NAME: ______________________ **Panther ID:** ____________

Exam 2 - MAC 2313 Fall 2017

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) Circle if each of the following statements is true or false and then give a brief justification of your answer:

(a) $\int_0^1 \int_{x^2}^x f(x,y)\,dy\,dx = \int_{x^2}^x \int_0^1 f(x,y)\,dx\,dy$ **True** **False**

Justification:

(b) If $z = z(x,y)$ and $x = x(u,v), y = y(u,v)$ then $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$ **True** **False**

Justification:

(c) The area of a region R in the xy-plane is given by $\int_R \int xy\,dA$ **True** **False**

Justification:

2. (8 pts) Set up an iterated double or triple integral to represent the volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 3 - x$.
Don't spend time evaluating the integral. It is not required for this one. Picture is required.

3. (8 pts) Find the tangent plane of the ellipsoid $x^2 + y^2 + 4z^2 = 12$ at the point $(2, 2, -1)$.

4. (12 pts) Compute $\int_R \int 2xy\, dA$, where R is the triangle bounded by $y = 3$, $y = -x + 1$ and $y = x + 1$.

5. (14 pts) Locate and classify all critical points of the function: $f(x, y) = -xy + y^3 + x^2$.

6. (20 pts) Consider the function $f(x, y) = \ln(1 + 2x + 3y)$.

(a) (6 pts) Compute $\dfrac{\partial f}{\partial x}(0, 0)$ and $\dfrac{\partial f}{\partial y}(0, 0)$.

(b) (6 pts) Given that $f(0, 0) = \ln 1 = 0$, use differentials (or local linear approximation) to estimate $f(1.02, 0.99)$.

(b) (8 pts) Find a unit vector for the direction in which f increases most rapidly at $(0, 0)$ and the rate of increase in this direction.

7. (10 pts) If the following limit exists, compute it. If the limit does not exist, justify why it doesn't.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

8. (16 pts) Use cylindrical or spherical coordinates to find the volume of the solid that is bounded inside the sphere $x^2+y^2+z^2=2$ and inside the cone $z=\sqrt{x^2+y^2}$. (Full computation required.)

9. (10 pts) Choose ONE:

(A) If the equation $f(x, y) = c$ defines y implicitly as a function of x, then at all points where $\partial f/\partial y \neq 0$,

$$\frac{dy}{dx} = \underline{\hspace{6cm}}$$

Fill in the blank with the appropriate formula and justify.

(B) Prove that for a differentiable function $f(x, y)$, the gradient is normal to the level curves of f.