NAME: ______________________________ **Panther ID:** ____________

Exam 2 - MAC 2313 Spring 2012

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (16 pts) Set up iterated double or triple integrals to represent each of the following. Don't spend time trying to evaluate the integrals. It is not required. (Picture is required in each case).

(a) (8 pts) The volume of the solid cut in the first octant by the plane $2x + 3y + z = 6$.

(b) (8 pts) The volume of the solid bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + z = 1$, $x = 0$, and $z = 0$.

2. (14 pts) Compute the value of the integral by first reversing the order of integration (picture required):

$$\int_0^2 \int_{y/2}^1 \cos(x^2)\, dx\, dy$$

3. (15 pts) Locate and classify all critical points of the function: $f(x, y) = x^4 + 2y^2 - 4xy$.

4. (21 pts) Given the function $f(x, y) = \ln(1 + x^2 + 4x - 3y)$ find:

(a) (7 pts) The tangent plane to the graph of the function at $(0, 0)$.

(b) (7 pts) The directional derivative of f at $(0, 0)$ in the direction of $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$.

(c) (7 pts) A unit vector in the direction in which f decreases most rapidly at $(0, 0)$ and the rate of change of the function in this direction.

5. (12 pts) Let $q = f(x, y, z)$ be a differentiable function of three variables. Suppose now that $x = u - v, y = v - w, z = w - u$, so that q can also be seen as a function of u, v, w by $q = f(u - v, v - w, w - u)$. Show that

$$\frac{\partial q}{\partial u} + \frac{\partial q}{\partial v} + \frac{\partial q}{\partial w} = 0.$$

6. (16 pts) Find the volume enclosed by $x^2 + y^2 + z^2 = a^2$ using cylindrical coordinates (or spherical coordinates, if you find it preferable). Obviously, you should get a familiar result. (Full computation required.)

7. (16 pts) A closed rectangular box must have a given volume of V_0 ft^3 and must be made from three different kinds of material. The top and the bottom are made from material costing p_1 cents per square foot, two opposite lateral sides are to be made from material costing p_2 cents per square foot and the other two opposite lateral sides are to be made from material costing p_3 cents per square foot. Find, in terms of p_1, p_2, p_3, V_0, the dimensions of the box that will minimize the cost.