Name: ____________________________ **Panther ID:** ______________

Midterm Exam MAT 3501 Fall 2016

1. (25 pts) For each of the following statements answer if it is True or False. Then give a **one line** justification of your answer. (2 pts answer, 3 pts justification)

(a) The only consecutive integers that are both prime are 2 and 3. **True False**
(Recall that 1, by definition, is not prime nor composite.)

Justification:

(b) $(x-2)$ is a factor of $p(x) = x^4 - 6x - 4$. **True False**

Justification:

(c) For all a, b rational numbers, a^b is rational. **True False**

Justification:

(d) For all integers l, m, n, if $l|(mn)$ then $l|m$ or $l|n$. **True False**

Justification:

(e) If a, b are integers and $gcd(a, b) = 1$, then $lcm(a, b) = ab$. **True False**

Justification:

(f) If p is prime and $p \geq 5$, then $(p+1) \,|\, p!$. **True False**

Justification:

2. (10 pts) Find **all** roots of the equation $x^3 + 2x + 3 = 0$. (Hint: the equation has a rational root.)

3. (10 pts) Let $x_1, x_2, x_3 \in \mathbf{C}$ be the roots of the polynomial $p(x) = 2x^3 + 3x + 1$. Use Viete's relations to find:

(a) $x_1 + x_2 + x_3$

(b) $x_1 x_2 x_3$

(c) $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}$

4. (16 pts) The product rule in Calculus states that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

(a) (6 pts) Show that there is a product rule for 3 functions and that it is

$$(f(x)g(x)h(x))' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

(b) (10 pts) Generalize the product rule for n functions and prove it by induction.

5. (10 pts) Show that, for any positive integer n, the greatest common divisor of $2n + 13$ and $n + 7$ is 1. As a consequence, justify that the fraction $\frac{n+7}{2n+13}$ is always in lowest terms.

6. (10 pts) Using mods, find the remainder of $2016^{2015} + 2015^{2016}$ when divided by 7.

7. (24 pts) Choose TWO of the following three (12 pts each)

(A) State and prove the Rational Root Theorem (it's OK if you give the detailed proof for just 1/2 of it).

(B) State and prove the quadratic formula.

(C) Prove that there are infinitely many primes (you can assume the prime factorization theorem).