Name: _____ **Panther ID:** _____

Worksheet - Sep. 22 MAT 3501 Fall 2016

1. (a) You are told that the equation $2x^4 + x^3 - x^2 + x - 3 = 0$ has two rational roots. Use the rational root theorem to make a short list of candidates and find at least one of the rational roots. (Not to spend too much time with computations, I'll tell you the other provided that your short list is fine.)

(b) Now use part (a) to find all roots (real or complex) of the equation $2x^4 + x^3 - x^2 + x - 3 = 0$

2. (a) Use the rational root theorem to show that $\sqrt[3]{5}$ is irrational.

(b) Use the rational root theorem to show that if n is not a perfect square then $\sqrt{n}$ is irrational.

3. (a) Do the multiplication on the right side to show the fundamental identity

$$A^n - B^n = (A - B)(A^{n-1} + A^{n-2}B + A^{n-3}B^2 + ... + AB^{n-2} + B^{n-1}) .$$

(b) Use the identity in (a) to show that if $k|n$ then $(x^k - 1)$ is a factor of $(x^n - 1)$.

(c) Show that if $k|n$ then $(10^k - 1)|(10^n - 1)$.

4. You can make a(nother) strong impression on your students by playing the following "divisibility prophecy" trick. Ask them to write two numbers (of different lengths) using only the digit 9. Then, using your "prophetic" powers, you will be able to tell them instantly whether or not the smaller number divides the larger one (and they can check you with a calculator). How do you do it? Do you think it works if you use a digit different than 9? Can you justify?