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## Worksheet - Nov. 10 - Geometry and Calculus

1. (Slightly modified version of the problem we did in class) Suppose that conical paper cups are made from disk sectors by gluing along the flat sides. Suppose we need conical paper cups that hold a given volume $V_{0}$. How should the paper cup be made in order to minimize the area of the paper used (assume there is no waste)? Show that the optimal paper cup will correspond to a certain optimal angle of the sector and find this optimal angle.
Note: This will give a quick way to test if an actual conical paper cup has minimal paper usage. You just cut it and compare the angle of the sector with the optimal one!
2. Find the dimensions of the cone of the smallest volume that can be circumscribed to a sphere of a given radius $r$.
3. (Slightly modified version of the James Bond problem on an earlier worksheet) James Bond is at the southernmost point of a circular lake. He needs to get to the northernmost point (diametrally opposite). It is known that James Bond runs twice as fast as he can swim and he can maintain his top speed (both for running and for swimming) for a long time. He can swim directly across the lake, he can run all the way around the lake, or he can try a combination of swimming and running. What path should James Bond take in order to minimize the time of the trip?
