Name: _____ **Panther ID:** _____

Exam 1 MAT 3501 Fall 2018

1. (15 pts) For each of the following, answer if the statement is True or False. Then give a one line justification of your answer. (2 pts answer, 3 pts justification)

(a) The product of any two irrational numbers is irrational. **True** **False**

Justification:

(b) If n is composite, then n has a divisor d so that $d \leq \sqrt{n}$. **True** **False**

Justification:

(c) In base 2, $\overline{111} + \overline{111} = \overline{1110}$. **True** **False**

Justification:

2. (15 pts) Find the prime factorization for each of the numbers 216 and 2016.
Then find gcd(216, 2016), lcm(216, 2016).

3. (15 pts) Prove (by induction, or otherwise) that for all $n \geq 0$, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

4. (15 pts) (a) (10 pts) Show that for a number written in base 10, $\overline{a_n a_{n-1} ... a_2 a_1 a_0}$, we have

$$\overline{a_n a_{n-1} ... a_2 a_1 a_0} \equiv a_0 - a_1 + a_2 - ... + (-1)^n a_n \pmod{11} \ .$$

(b) (5 pts) Apply part (a) to find the remainder of 987654321 when divided by 11.

5. (15 pts) Show that if the prime decomposition of a number is $N = p_1^{n_1} p_2^{n_2} ... p_k^{n_k}$, with $p_1, p_2, ..., p_k$ distinct primes, then the number of divisors of N (including 1 and N) is given by $(n_1 + 1)(n_2 + 1)...(n_k + 1)$.

6. (15 pts) Describe all integer solutions (if any) of the Diophantine equation $9x + 7y = 5$.

7. (15 pts) Choose ONE of the following proofs. If you do both, the second score may give some bonus towards a previous problem with a lower score.

(A) Show that if a, b are positive integers, then there exist integers m, n so that $ma + nb = \gcd(a, b)$.

(B) Show that if p is a prime number and $p|(ab)$ then $p|a$ or $p|b$. (You may use the result in (B) for proving (C).)