Name: ______________________ **Panther ID:** ____________

Worksheet - Oct. 16 MAT 3501 Fall 2018

1. (a) Use Euler's formula to discover identities for $\cos(x+y)$ and $\sin(x+y)$. Replacing y by $-y$, you will also find the identities for $\cos(x-y)$ and $\sin(x-y)$.

(b) Use Euler's formula to justify DeMoivre's identity:

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

(c) Use DeMoivre's identity to find a formula for $\cos(5\theta)$ in terms of $\cos(\theta)$.

2. (a) Use trigonometric identities in Exercise 1(a), to show the identity:

$$\cos(n+1)\theta = 2\cos n\theta\cos\theta - \cos(n-1)\theta$$

(b) Use part (a) and induction to show that for any θ and any positive integer n, there exists a polynomial with integer coefficients and lead coefficient 1 so that

$$2\cos n\theta = 1x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0, \text{ where } x = 2\cos\theta\ .$$

In other words, we can represent $2\cos n\theta$ as a polynomial of degree n with integer coefficients and lead coefficient 1 in x, where $x = 2\cos\theta$.

(c) Use (b) to show that if θ is a rational number representing an angle in degrees, then $\cos(\theta)$ is an algebraic number.

(d) Suppose now that θ is an integer representing an angle in degrees, $0 \le \theta \le 90$. Show that if θ is not 0, 90, or 60, then $\cos(\theta)$ must be irrational. *Hint:* Use (b) or (c) and the rational root Theorem.

3. (a) Consider the function $f : \mathbf{C} \to \mathbf{C}$ defined by $f(z) = 3z$. Describe geometrically this function; that is describe geometrically, the relation between z and $f(z)$. In general, functions $f : \mathbf{C} \to \mathbf{C}$ of the form $f(z) = kz$, with $k \in \mathbf{R}$ are called homotheties of the plane.

(b) Consider the function $f : \mathbf{C} \to \mathbf{C}$ defined by $f(z) = z/(1-i)$. Describe geometrically this function; that is describe geometrically, the relation between z and $f(z)$. *Hint:* Think of the polar form of z and $f(z)$.