Name:

## Worksheet - Nov. 15

## 1. Important take home problem (20 points value): Due Tuesday, Nov. 20.

(a) Review the important points (and lines) related to the geometry of a triangle. Write the definition of circumcenter (usually denoted $O$ ), incenter (usually denoted $I$ ), centroid (usually denoted $G$ ) and orthocenter (usually denoted $H$ ) of an arbitrary triangle.
(b) If don't have this already, go to https://www.geogebra.org/ and download GeoGebra in your computer, or your phone/tablet. Play a bit with its features.
(c) Construct, using GeoGebra, the circumcircle of a given triangle. There is a direct tool for this, but for your practice start with the vertices and do the rest of the steps. Moreover, the direct tool does not show the position of the circumcenter. Submit a printout of your work, or show your work on your phone or tablet Next, use the "move" feature of the program (the arrow on the left) to move one of the vertices of the triangle keeping the other two vertices fixed. Notice what happens with the position of the circumcenter versus the triangle. Formulate a conjecture of the type: For "these" triangles the cicumcenter will lie inside the triangle, for "these" triangles the circumcenter will lie outside of the triangle and for "these" triangles the cicumcenter will lie on one side the triangle (be even more specific in this case). You'd get bonus point if you prove your conjecture.
(d) Construct, using GeoGebra, the circumcircle, $O$, the orthocenter, $H$, and the centroid of a triangle, $G$. Draw the line joining $O$ and $H$ and observe the position of $G$ versus this line. Use the "move" feature of the program (the arrow on the left) and formulate a conjecture about the position of $H, O, G$ in an arbitrary triangle. This is the so called Euler line of a triangle. There is also a certain (constant) ratio of the distances $|H G|,|G O|$, that you can discover.
(e) (bonus) Prove (or look up a proof and understand it for) the conjectures in part (d).

