Diagnostic Test – Calculus 1 - Spring 2015 NAME: Solution & Grading Key

1. (6 pts) Fill in the exact values: (1pt each – no partial credit)

$\log_3 81 =$ 4 since $3^4 = 81$ $e^{2\ln 3} = e^{\ln(3^2)} = 9$ $\arctan(1) = \frac{\pi}{4}$ {since $\tan(\frac{\pi}{4}) = 1$}

$\left(\frac{100}{9}\right)^{-1/2} = \left(\frac{9}{100}\right)^{\frac{1}{2}} = \sqrt{\frac{9}{100}} = \frac{3}{10}$ $\sin(\frac{7\pi}{6}) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$ $\cos 0 =$ 1

2. (4 pts) Circle the correct answer (assume that $x \neq 0$): (2pts each, no partial credit)

(a) The expression $\dfrac{3x^2}{x^4 + 9x^2}$ is equivalent with: $= \dfrac{3x^2}{x^2(x^2+9)} = \dfrac{3}{x^2+9}$

(i) $\dfrac{1}{x^2+3}$ (ii) $\dfrac{3}{x^2} + \dfrac{1}{3}$ (iii) $\dfrac{1}{x^4+3}$ (iv) $\dfrac{3}{x^2+9}$ (v) $\dfrac{3}{10x^2}$

(b) The expression $\dfrac{x^2}{\sqrt[3]{x^2}}$ is equivalent with: both are equivalent to $x^{\frac{4}{3}}$

(i) $\sqrt{x}$ (ii) 1 (iii) $x\sqrt[3]{x}$ (iv) $x^{-1/3}$ (v) none of the above

3. (4 pts) (a) (1 pts) The domain of $f(x) = -1 + \sqrt{x+4}$ is $x \geq -4$ or $x \in [-4, +\infty)$.

(b) (3 pts) For $f(x) = -1 + \sqrt{x+4}$, determine the formula of its inverse function $f^{-1}(x)$. (2 pts algebra) (1pt final answer)

$y = -1 + \sqrt{x+4}$

$\Rightarrow y+1 = \sqrt{x+4}$

$\Rightarrow (y+1)^2 = x+4$

$x = (y+1)^2 - 4$

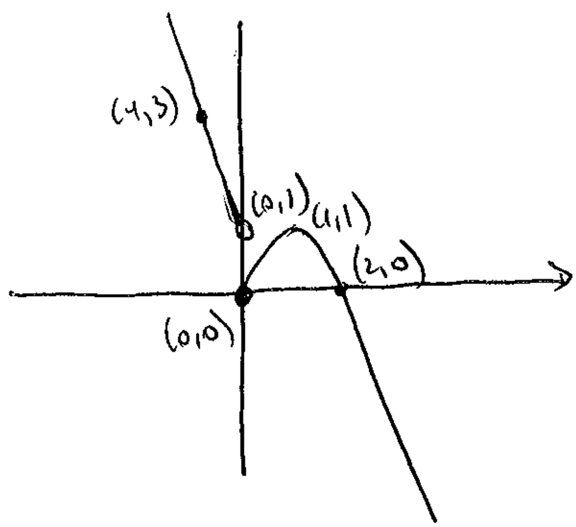
swap x & y, so $y = f^{-1}(x) = (x+1)^2 - 4$

4. (4 pts) Sketch the graph of the function below.
Write the coordinates of axis intercepts.

$$g(x) = \begin{cases} -2x+1 & \text{if } x < 0 \\ 2x - x^2 & \text{if } x \geq 0 \end{cases}$$

1,5 pts for each piece of the graph

1 pt for the intercepts



5. (6 pts) In each case, circle "True" or "False".

If $f(x) = 2 - 3x$, then $f(x+4) = 2 - 3x + 4 = 6 - 3x$ True False

Let $f(x)$ be an invertible function with inverse $f^{-1}(x)$. If $f(5) = 5$, then $f^{-1}(5) = \frac{1}{5}$ True False

For all $x > 0, y > 0$, $\sqrt{x^2 + y^2} = x + y$ True False

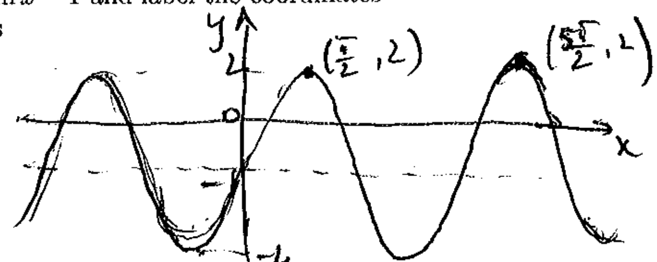
For all $x > 0, y > 0$, $\log(xy) = \log x + \log y$ True False

For all real x, $\dfrac{x+2}{x^2+4} = \dfrac{1}{x+2}$ True False

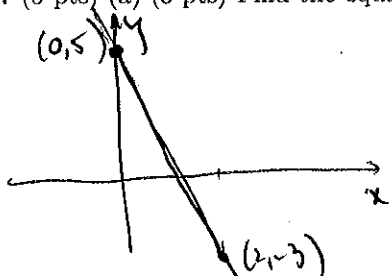
For all real x, $\cos^2 x = 1 - \sin^2 x$ True False

6. (3 pts) Sketch the graph of $y = 3\sin x - 1$ and label the coordinates of at least two of the maximum points (that is, points where y is maximum).

2pts for graph
1pt for coordinates



7. (5 pts) (a) (3 pts) Find the equation of the line that contains the points $(0, 5)$ and $(2, -3)$.



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{2-0} = -4 \quad (1.5\text{pts})$$

$$\boxed{y - 5 = -4(x - 0)} \quad (1.5\text{pts})$$

or $\boxed{y = -4x + 5}$

(b) (2 pts) Find the equation of the circle with center at $(2, 0)$ and with radius 2.

$$(x-2)^2 + y^2 = 2^2$$

8. (3 pts) If $f(x) = 3x - x^2$, compute and simplify the expression $\frac{f(2+h) - f(2)}{h}$.

$f(2) = 3 \cdot 2 - 2^2 = 2$ (0.5pts)

$f(2+h) = 3 \cdot (2+h) - (2+h)^2$ (0.5pts)

$$\frac{f(2+h) - f(2)}{h} = \frac{3 \cdot (2+h) - (2+h)^2 - 2}{h} = \frac{\cancel{6} + 3h \cancel{- 4} - 4h - h^2 \cancel{- 2}}{h} =$$

$$= \frac{-h - h^2}{h} = \frac{-h(1+h)}{h} = \boxed{-(1+h)}$$

(2pts for the algebra)

9. (10 pts) Find all real solutions of the following equations (2 pts each):

(a) $x^4-3x^2-4=0$

$$(x^2-4)(x^2+1)=0$$
$$(x-2)(x+2)(x^2+1)=0$$

the real solutions are $\boxed{x=2,\ x=-2}$

Note that $x^2+1=0$ has complex (non real) solutions $x=\pm\sqrt{-1}=\pm i$

(b) $2x^{4/3}-x=0$

$$x(2x^{\frac{1}{3}}-1)=0 \qquad x=0 \text{ or } 2x^{\frac{1}{3}}-1=0$$
$$x^{\frac{1}{3}}=\frac{1}{2} \Rightarrow x=\left(\frac{1}{2}\right)^3=\frac{1}{8}$$

so $\boxed{\begin{matrix} x=0 \\ x=\frac{1}{8}\end{matrix}}$

(c) $2\cos^2 x=1$ OK to find all solutions $x\in[0,2\pi]$ for this one.

$$\cos^2 x=\frac{1}{2} \Rightarrow \cos x=\pm\frac{1}{\sqrt{2}} \qquad x=\frac{\pi}{4},\ x=\frac{3\pi}{4},\ x=\frac{5\pi}{4},\ x=\frac{7\pi}{4}$$

(d) $5\cdot(2^{3x})=7$ Leave your answer as a logarithm for this one.

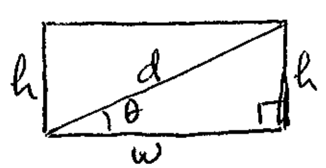
$$2^{3x}=\frac{7}{5} \Rightarrow 3x=\log_2\left(\frac{7}{5}\right) \quad \text{so } x=\frac{\log_2\left(\frac{7}{5}\right)}{3}=\frac{\log_2(7)-\log_2(5)}{3}$$
$$\text{or } \ln(2^{3x})=\ln\left(\frac{7}{5}\right) \Rightarrow 3x\ln 2=\ln\left(\frac{7}{5}\right) \Rightarrow x=\frac{\ln 7-\ln 5}{3\ln 2}$$

(e) $ax^2+bx+c=0$ I want to check you know the quadratic formula. When are the solutions real?

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

10. (5 pts) For a 16:9 widescreen TV, the ratio (width of screen)/(height of screen) is 16/9.

(a) (2 pts) For a 16:9 TV, what is the angle that the diagonal is making with the horizontal? Leave your answer as an inverse trigonometric function.



Given $\frac{w}{h}=\frac{16}{9}$

From the right triangle, $\tan\theta=\frac{h}{w}=\frac{9}{16}$

Thus $\theta=\arctan\left(\frac{9}{16}\right)$

(b) (3 pts) For a 16:9 TV, find a function expressing the area of the screen, A, in terms of its diagonal length d.

$$A=w\cdot h=\left(\frac{16}{9}h\right)\cdot h=\frac{16}{9}h^2 \longrightarrow A=\frac{16}{9}\cdot\frac{9^2}{9^2+16^2}d^2$$

$$\boxed{A=\frac{16\cdot 9}{9^2+16^2}d^2}$$

we need to express h^2 in terms of d.

By Pythagora $w^2+h^2=d^2$

We substitute $w=\frac{16}{9}h$

$$\Rightarrow \left(\frac{16}{9}\right)^2h^2+h^2=d^2$$

$$\Rightarrow h^2\left(1+\frac{16^2}{9^2}\right)=d^2 \Rightarrow h^2\cdot\frac{9^2+16^2}{9^2}=d^2 \Rightarrow h^2=\frac{9^2}{9^2+16^2}d^2$$