**Diagnostic Test – Calculus 1** - Spring 2015          **NAME:** _____

**1.** (6 pts) Fill in the exact values:

   $\log_3 81 =$                    $e^{2\ln 3} =$                    $\arctan(1) =$

   $\left(\dfrac{100}{9}\right)^{-1/2} =$                    $\sin(\dfrac{7\pi}{6}) =$                    $\cos 0 =$

**2.** (4 pts) Circle the correct answer (assume that $x \neq 0$):

(a) The expression $\dfrac{3x^2}{x^4 + 9x^2}$ is equivalent with:

   $(i)\ \ \dfrac{1}{x^2 + 3}$          $(ii)\ \ \dfrac{3}{x^2} + \dfrac{1}{3}$          $(iii)\ \ \dfrac{1}{x^4 + 3}$          $(iv)\ \ \dfrac{3}{x^2 + 9}$          $(v)\ \ \dfrac{3}{10x^2}$

(b) The expression $\dfrac{x^2}{\sqrt[3]{x^2}}$ is equivalent with:

   $(i)\ \ \sqrt{x}$          $(ii)\ \ 1$          $(iii)\ \ x\sqrt[3]{x}$          $(iv)\ \ x^{-1/3}$          $(v)\ \$ none of the above

**3.** (4 pts) (a) (2 pts) The domain of $f(x) = -1 + \sqrt{x + 4}$ is _____ .

(b) (2 pts) For $f(x) = -1 + \sqrt{x + 4}$, determine the formula of its inverse function $f^{-1}(x)$.

**4.** (4 pts) Sketch the graph of the function below.
Write the coordinates of axis intercepts.

$g(x) = \begin{cases} -2x + 1 & \text{if } x < 0 \\ 2x - x^2 & \text{if } x \geq 0 \end{cases}$

**5.** (6 pts) In each case, circle "True" or "False".

If $f(x) = 2 - 3x$, then $f(x+4) = 2 - 3x + 4 = 6 - 3x$          True     False

Let $f(x)$ be an invertible function with inverse $f^{-1}(x)$. If $f(5) = 5$, then $f^{-1}(5) = \frac{1}{5}$      True     False

For all $x > 0, y > 0,$   $\sqrt{x^2 + y^2} = x+y$         True     False

For all $x > 0, y > 0,$   $\log(xy) = \log x + \log y$         True     False

For all real $x$,   $\dfrac{x+2}{x^2+4} = \dfrac{1}{x+2}$         True     False

For all real $x$,   $\cos^2 x = 1 - \sin^2 x$         True     False

**6.** (3 pts) Sketch the graph of $y = 3\sin x - 1$ and label the coordinates
of at least two of the maximum points
(that is, points where $y$ is maximum).

**7.** (5 pts) (a) (3 pts) Find the equation of the line that contains the points $(0, 5)$ and $(2, -3)$.

(b) (2 pts) Find the equation of the circle with center at $(2, 0)$ and with radius 2.

**8.** (3 pts) If $f(x) = 3x - x^2$, compute and simplify the expression $\frac{f(2+h) - f(2)}{h}$.

**9.** (10 pts) Find all real solutions of the following equations (2 pts each):

(a)  $x^4 - 3x^2 - 4 = 0$

(b)  $2x^{4/3} - x = 0$

(c)  $2\cos^2 x = 1$                    OK to find all solutions $x \in [0, 2\pi]$ for this one.

(d)  $5 \cdot (2^{3x}) = 7$              Leave your answer as a logarithm for this one.

(e)  $ax^2 + bx + c = 0$       I want to check you know the quadratic formula. When are the solutions real?

**10.** (5 pts) For a 16:9 widescreen TV, the ratio (width of screen)/(height of screen) is 16/9.

(a) (2 pts) For a 16:9 TV, what is the angle that the diagonal is making with the horizontal? Leave your answer as an inverse trigonometric function.

(b) (3 pts) For a 16:9 TV, find a function expressing the area of the screen, $A$, in terms of its diagonal length $d$.