Name: Solution Key Panther ID: ____________

**Worksheet week 10** Calculus I Spring 2015

1. Use l'Hopital's rule to compute each of the following limits (2.5 pts each):

(a) $\lim_{x\to 0} \frac{\cos(5x)-\cos(3x)}{x^2} \underset{l'H}{\overset{\frac{0}{0}}{=}}$

$= \lim_{x\to 0} \frac{-5\sin(5x)+3\sin(3x)}{2x} \underset{l'H}{\overset{\frac{0}{0}}{=}}$

$= \lim_{x\to 0} \frac{-25\cos(5x)+9\cos(3x)}{2} =$

$= \frac{-25\cdot 1+9\cdot 1}{2} = \boxed{-8}$

(b) $\lim_{x\to+\infty} \frac{(\ln x)^2}{x} \underset{l'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to+\infty} \frac{2(\ln x)\cdot\frac{1}{x}}{1}$

$= \lim_{x\to+\infty} \frac{2\ln x}{x} \underset{l'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to+\infty} \frac{2\cdot\frac{1}{x}}{1}$

$= \boxed{0}$

(c) $\lim_{x\to 0} \left(\cot x - \frac{1}{x}\right)$ $\infty - \infty$ indeterminate form

$= \lim_{x\to 0} \left(\frac{\cos x}{\sin x} - \frac{1}{x}\right) =$

$= \lim_{x\to 0} \frac{x\cos x - \sin x}{x\cdot\sin x} \underset{l'H}{\overset{\frac{0}{0}}{=}}$

$= \lim_{x\to 0} \frac{1\cdot\cancel{\cos x} + x\cdot(-\sin x) - \cancel{\cos x}}{1\cdot\sin x + x\cdot\cos x}$

$= -\lim_{x\to 0} \frac{x\cdot\sin x}{\sin x + x\cdot\cos x} =$

$= -\lim_{x\to 0} \frac{\cancel{x}\cdot\sin x}{\cancel{x}\left(\frac{\sin x}{x}+\cos x\right)} = \frac{0}{1+1} = \boxed{0}$

factor x by force in denom.

you could continue here with l'H one more time

(d) $\lim_{x\to 0^+} \left(1+\sin(4x)\right)^{\cot x} =$ $1^\infty$ indet. form.

$= \lim_{x\to 0^+} e^{\ln\left(\left(1+\sin(4x)\right)^{\cot x}\right)} =$

$= e^{\lim_{x\to 0^+} \cot x\cdot\ln(1+\sin(4x))} = \boxed{e^4}$

$\lim_{x\to 0^+} \cot x\cdot\ln(1+\sin(4x)) \overset{\infty\cdot 0}{=}$

$= \lim_{x\to 0^+} \frac{\ln(1+\sin(4x))}{\tan x} \underset{l'H}{\overset{\frac{0}{0}}{=}}$

$= \lim_{x\to 0^+} \frac{\frac{1}{1+\sin(4x)}\cdot\cos(4x)\cdot 4}{\sec^2(x)}$

$= \frac{\frac{1}{1+0}\cdot 1\cdot 4}{1} = 4$