NAME: Solution Key **Panther ID:** ________

Quiz 2 - MAC 2311, Summer B 2019

To receive full credit you MUST SHOW ALL YOUR WORK.

1. (10 pts) Compute $\frac{dy}{dx}$ for each of the following functions. Simplify your answer when possible. (2 pts each)

(a) $y = 3x^5 - \frac{1}{2x^2} = 3x^5 - \frac{1}{2}x^{-2}$

$$\frac{dy}{dx} = 15x^4 - \frac{1}{2}\cdot(-2)x^{-3}$$

$$\frac{dy}{dx} = \boxed{15x^4 + \frac{1}{x^3}}$$

(b) $y = x^2e^{-3x}$

Product Rule

$$\frac{dy}{dx} = \left(x^2e^{-3x}\right)' = (x^2)'e^{-3x} + x^2\left(e^{-3x}\right)'$$

$$= 2xe^{-3x} + x^2\cdot e^{-3x}\cdot(-3)$$

from Chain Rule

$$= 2xe^{-3x} - 3x^2e^{-3x}$$

$$\boxed{\frac{dy}{dx} = x\cdot e^{-3x}(2-3x)}$$

(c) $y = \ln(\sec x + \tan x)$

$$\frac{dy}{dx} = \left(\ln(\sec x + \tan x)\right)' =$$

$$= \frac{1}{\sec x + \tan x}\cdot(\sec x + \tan x)' =$$

Chain Rule

$$= \frac{\sec x\cdot\tan x + \sec^2 x}{\sec x + \tan x} =$$

$$= \frac{\sec x\,\cancel{(\tan x + \sec x)}}{\cancel{\sec x + \tan x}} = \boxed{\sec x}$$

(d) $y = \sin^4(\sqrt{x}) = \left(\sin\left(x^{\frac{1}{2}}\right)\right)^4$

Chain Rule

$$\frac{dy}{dx} = \left(\left(\sin\left(x^{\frac{1}{2}}\right)\right)^4\right)' =$$

$$= 4\left(\sin\left(x^{\frac{1}{2}}\right)\right)^3\cdot\left(\sin\left(x^{\frac{1}{2}}\right)\right)'$$

$$= 4\sin^3(\sqrt{x})\cdot\cos\left(x^{\frac{1}{2}}\right)\cdot\left(x^{\frac{1}{2}}\right)'$$

Chain Rule again

$$= \cancel{4}^{2}\sin^3(\sqrt{x})\cdot\cos(\sqrt{x})\cdot\frac{1}{\cancel{2}}x^{-\frac{1}{2}} =$$

$$= \boxed{\frac{2\sin^3(\sqrt{x})\cdot\cos(\sqrt{x})}{\sqrt{x}}}$$

(e) $y = (\ln x)^x$ Hint: Use logarithmic differentiation for this one.

$$\ln y = \ln\left((\ln x)^x\right)$$

$$\ln y = x\cdot\ln(\ln x) \quad \Big|\ \frac{d}{dx}$$

$$(\ln y)' = (x\cdot\ln(\ln x))'$$

$$\frac{1}{y}\cdot y' = 1\cdot\ln(\ln x) + \cancel{x}\cdot\frac{1}{\ln x}\cdot\frac{1}{\cancel{x}}$$

$$y' = y\left(\ln(\ln x) + \frac{1}{\ln x}\right)$$

$$\text{so } \frac{dy}{dx} = (\ln x)^x\cdot\left[\ln(\ln x) + \frac{1}{\ln x}\right]$$