MAP 2302: Worksheet July 23, 2018 **Group nr:** ______ **Names:** Solution key

1. Use Laplace transform to solve the IVP

$$y''+5y'+6y = h(t)\ ,\ \ y(0)=0,\ \ y'(0)=0, \quad \text{where } h(t) = \begin{cases} 6, & 0<t<2 \\ 0, & t>2 \end{cases}$$

$$\mathcal{L}(y'') + 5\,\mathcal{L}(y') + 6\,\mathcal{L}(y) = \mathcal{L}(h(t)) \quad (*)$$

denote $\mathcal{L}(y) = Y(s)$

since $y(0)=0$, $y'(0)=0$

$$\mathcal{L}(y'') = s^2 Y \quad \text{and} \quad \mathcal{L}(y') = sY \qquad (1)$$

Compute $\mathcal{L}(h(t))$ by observing that

$h(t) - 6u_0(t) = -6u_2(t)$ so $h(t) = 6u_0(t) - 6u_2(t)$

$$\mathcal{L}(h(t)) = 6\,\mathcal{L}(u_0) - 6\,\mathcal{L}(u_2) = \frac{6}{s} - \frac{6e^{-2s}}{s} \qquad (2)$$

Replacing (1) and (2) in $(*)$ we get

$$(s^2+5s+6)Y = \frac{6}{s} - \frac{6e^{-2s}}{s} \quad \text{or}$$

$$Y(s) = \frac{6}{s(s+2)(s+3)} - \frac{6e^{-2s}}{s(s+2)(s+3)}$$

By partial fractions $\dfrac{6}{s(s+2)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$ algebra $A=1,\ B=-3,\ C=2$

Thus $y(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{3}{s+2} + \frac{2}{s+3}\right) - \mathcal{L}^{-1}\left(\frac{e^{-2s}}{s}\right) + 3\mathcal{L}^{-1}\left(e^{-2s}\cdot\frac{1}{s+2}\right) - 2\mathcal{L}^{-1}\left(e^{-2s}\cdot\frac{1}{s+3}\right)$

$$y(t) = 1 - 3e^{-2t} + 2e^{-3t} - u_2(t) + 3u_2(t)\cdot e^{-2(t-2)} - 2u_2(t)\,e^{-3(t-2)}$$

or

$$y(t) = \begin{cases} 1 - 3e^{-2t} + 2e^{-3t} & \text{if } 0<t<2 \\ \cancel{1} - 3e^{-2t} + 2e^{-3t} \cancel{-1} + 3e^{-2t+4} - 2e^{-3t+6} & \text{if } t>2 \end{cases}$$