Midterm - MAD 2104 - Summer A 2014 NAME: ______________________

To receive credit you MUST SHOW ALL YOUR WORK.

1. (10 pts) Show that $(p \to r) \wedge (q \to r)$ is logically equivalent with $(p \vee q) \to r$.

2. (15 pts) Consider the relation R on the set of all integers $\mathbf{Z}$ defined by

$$(x, y) \in R \text{ if and only if } (x - y = 0 \text{ or } x - y = 1 \text{ or } x - y = -1) .$$

Answer "Yes" or "No" (2 pts) and briefly justify (1 pt). Correct answer with wrong justification receives only 1 pt.

(a) Is R reflexive?

(b) Is R symmetric?

(c) Is R antisymmetric?

(d) Is R transitive?

(e) Is R an equivalence relation on $\mathbf{Z}$?

3. (8 pts) Write, in simple English, the negation of each of the following statements. Using logical connectors/quantifiers as an intermediate step may help. Do not start with a negation like "It is not true that ..."

(a) If John is enrolled at FIU then he has an FIU e-mail account.

(b) There is a problem in this exam that nobody in the class will solve correctly.

4. (16 pts) Suppose A and B are arbitrary sets.

(a) (8 pts) Show that $A - (A - B) = A \bigcap B$.

(b) (8 pts) Prove or disprove: If $\mathcal{P}(\mathcal{A}) \subseteq \mathcal{P}(\mathcal{B})$ then $A \subseteq B$.

5. (8 pts) Let $P(m,n)$ the statement "m divides n", where the domain for both variables consists of all positive integers. (By "m divides n" we mean that $n = km$ for some integer k.) Determine the truth value of each of the following. No justification needed, just the answer will be graded.

(a) $P(5,23)$

(b) $\forall m \exists n\ P(m,n)$

(c) $\exists n \forall m\ P(m,n)$

(d) $\exists m \forall n\ P(m,n)$

6. (14 pts) Decide whether is possible or not to cover each of the following with standard 2×1 dominoes. Justify your answer.

(a) (7 pts) A 3×4 checkerboard with two opposite corners removed.

(b) (7 pts) A 4×4 checkerboard with two opposite corners removed.

7. (10 pts) On the island of Smullyan there are two types of inhabitants, knights, who always tell the truth, and knaves, who always lie. You are on this island and meet two inhabitants A and B. A says "I am a knave or B is a knight" and B says nothing. Determine, if possible, what are A and B. Briefly describe your reasoning.

8. (16 pts) Determine if each of the following statements is True or False. No justification needed, just the answer will be graded.

(a) The contrapositive of $p \to q$ is $\neg p \to \neg q$.

(b) For any set A, $\emptyset \subseteq A$.

(c) For any set A, $\emptyset \in A$.

(d) For any two sets A and B, $A \times B = B \times A$.

(e) $\forall x(P(x) \vee Q(x)) \equiv (\forall x P(x)) \vee (\forall x Q(x))$

(f) The set $S = \{n^2 \mid n \in \mathbf{N}\}$ is countable.

(g) The set of irrational numbers in the interval $(0, 1)$ is countable.

(h) The function $f(x) = x^2 + 1$ is a bijection from $\mathbf{R}$ to $\mathbf{R}$.

9. Choose ONE:

(a) (10 pts) Prove or disprove: The sum of a rational number with an irrational number is irrational.

(b) (14 pts) Show that the set of infinite bit strings is uncountable.