Name: Solution Key PanthID: ________________

Quiz 2-A MAD 2104 Summer A 2015

1. (6 pts) For each of the following, circle the correct answer. Only one answer is correct. No proof or justification necessary, but a Venn diagram may help you.

(a) For any sets S, T, if $S \subseteq T$ then:

(i) $\overline{T} \subseteq \overline{S}$ (ii) $T \subseteq S \cap T$ (iii) $T - S = \emptyset$ (iv) $S \cap T = S \cup T$

(b) According to DeMorgan's laws $\overline{A \cup (B \cap C)} =$

(i) $\overline{A} \cup (\overline{B} \cap \overline{C})$ (ii) $\overline{A} \cap (B \cap C)$ (iii) $\overline{A} \cap (\overline{B} \cup \overline{C})$ (iv) $A \cap (B \cup C)$

(c) Which of the following is true for all sets A and B?

(i) $A \cup \overline{B} = \overline{A \cap B}$ (ii) $A \cup \overline{B} = (A \cap B) \cup B$ (iii) $(A \cup B) \cap B = B$ (iv) $A - (A - B) = B$

2. (7 pts) The symmetric difference of two sets A, B is denoted by $A \oplus B$ and is defined sometimes by $(A - B) \cup (B - A)$ and sometimes by $(A \cup B) - (A \cap B)$.

(a) (6 pts) Show that for any sets A and B, $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) (1 pts) To what logical connector does the set operation $A \oplus B$ correspond to? <u>exclusive or</u>

Sol. 1 – using logic

(a) $x \in (A \setminus B) \cup (B \setminus A) \iff x \in A \setminus B \vee x \in B \setminus A \iff$

$\iff (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A) \overset{\text{distr. law}}{\iff} (x \in A \vee (x \in B \wedge x \notin A)) \wedge (x \notin B \vee (x \in B \wedge x \notin$

$\overset{\text{distr. law}}{\iff} \Big[(x \in A \vee x \in B) \wedge \underbrace{(x \in A \vee x \notin A)}_{\text{tautology}}\Big] \wedge \Big[\underbrace{(x \notin B \vee x \in B)}_{\text{tautology}} \wedge (x \notin B \vee x \notin A)\Big]$

$\iff [x \in A \vee x \in B] \wedge [\neg (x \in B \wedge x \in A)] \iff x \in A \cup B \wedge \neg (x \in A \cap B)$

DeMorgan

$\iff x \in A \cup B \wedge x \notin A \cap B \iff x \in (A \cup B) \setminus (A \cap B)$

Thus $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

<u>Solution 2</u> using set identities

$(A \setminus B) \cup (B \setminus A) = (A \cap \overline{B}) \cup (B \cap \overline{A}) = [A \cup (B \cap \overline{A})] \cap [\overline{B} \cup (B \cap \overline{A})]$

$= [(A \cup B) \cap (A \cup \overline{A})] \cap [(\overline{B} \cup B) \cap (\overline{B} \cup \overline{A})] = [(A \cup B) \cap U] \cap [U \cap \overline{B \cap A}$

universe

$= (A \cup B) \cap \overline{A \cap B} = (A \cup B) - (A \cap B)$

3. (8 pts) For each of the following, circle **all** the true statements. More than one, or none, of the answers could be true. No proof or justification necessary.

(a) Let $A = \{1, 2, 3, 4\}$ and let $\mathcal{P}(A)$ be the power set of A. From the statements below, circle the ones which are true:

(*i*) $2 \in \mathcal{P}(A)$ (*ii*) $\{1, 3\} \subseteq \mathcal{P}(A)$ (*iii*) $\{1, 3\} \in \mathcal{P}(A)$ (*iv*) $\{\{2\}, \{4\}\} \subseteq \mathcal{P}(A)$

(b) On the set of all people, let $\mathcal{R}$ be the relation given by $(a, b) \in \mathcal{R}$ if and only if a and b have the same birthday (day and month). From the statements below, circle the ones which are true:

(*i*) $\mathcal{R}$ is reflexive (*ii*) $\mathcal{R}$ is symmetric (*iii*) $\mathcal{R}$ is anti-symmetric (*iv*) $\mathcal{R}$ is transitive

4. (6 pts) If possible, give an example of a relation $\mathcal{R}$ on the set $A = \{a, b, c\}$ which is reflexive, anti-symmetric, but not symmetric and not transitive. If not possible, explain why.

One of the possible examples:

$R = \{(a,a), (b,b), (c,c), (a,b), (b,c)\}$

Name: Solution Key PanthID: ____________

Quiz 2-B MAD 2104 Summer A 2015

1. (6 pts) For each of the following, circle the correct answer. Only one answer is correct. No proof or justification necessary, but a Venn diagram may help you.

(a) For any sets S, T, if $S \subseteq T$ then:

(*i*) $\overline{S} \subseteq \overline{T}$ (*ii*) $S \cap T = \emptyset$ (*iii*) $S - T = \emptyset$ (*iv*) $S \cap T = S \cup T$

(b) According to DeMorgan's laws $\overline{A \cap (B \cup C)} =$

(*i*) $\overline{A} \cap (\overline{B} \cup \overline{C})$ (*ii*) $\overline{A} \cup (\overline{B} \cap \overline{C})$ (*iii*) $\overline{A} \cup (B \cup C)$ (*iv*) $A \cup (B \cap C)$

(c) Which of the following is true for all sets A and B?

(*i*) $A \cap \overline{B} = \overline{A \cap B}$ (*ii*) $A \cap \overline{B} = A - (A \cap B)$ (*iii*) $(A \cup B) - B = A$ (*iv*) $A \cup B = A \cap B$

2. (7 pts) The symmetric difference of two sets A, B is denoted by $A \oplus B$ and is defined sometimes by $(A - B) \cup (B - A)$ and sometimes by $(A \cup B) - (A \cap B)$.

(a) (6 pts) Show that for any sets A and B, $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) (1 pts) To what logical connector does the set operation $A \oplus B$ correspond to?

see solution on Quiz 2-A

3. (8 pts) For each of the following, circle **all** the true statements. More than one, or none, of the answers could be true. No proof or justification necessary.
(a) Let $A = \{1, 2, 3, 4\}$ and let $\mathcal{P}(A)$ be the power set of A. From the statements below, circle the ones which are true:

(*i*) $\{2, 4\} \in \mathcal{P}(A)$ (*ii*) $\{\{2\}, \{4\}\} \subseteq \mathcal{P}(A)$ (*iii*) $\{1, 3\} \subseteq \mathcal{P}(A)$ (*iv*) $1 \in \mathcal{P}(A)$

(b) On the set of all people, let $\mathcal{R}$ be the relation given by $(a, b) \in \mathcal{R}$ if and only if a and b have the same birthday (day and month). From the statements below, circle the ones which are true:

(*i*) $\mathcal{R}$ is reflexive (*ii*) $\mathcal{R}$ is symmetric (*iii*) $\mathcal{R}$ is anti-symmetric (*iv*) $\mathcal{R}$ is transitive

4. (6 pts) If possible, give an example of a relation $\mathcal{R}$ on the set $A = \{a, b, c\}$ which is symmetric, transitive, but not reflexive and not anti-symmetric. If not possible, explain why.

One of the possible examples

$R = \{(a,b), (b,a), (a,a), (b,b)\}$