

Name: Solution Key

PanthID: _____

Quiz 5 - v.A

MAD 2104

Summer A 2015

Each of the following problems is worth 3 pts. It's OK if you just give the answers, but if the answer is wrong, I can give you some partial credit only if you provide some justification which gets close to the solution of the problem (justification can be short - e.g. complement rule, addition rule, etc.)

1. How many bit strings of length seven are there?

$$2^7$$

2. How many bit strings of length seven start with a 1 and end with a 0?

$$2^5 \quad \underline{1 \quad \quad \quad 0} \rightarrow \text{five free bits}$$

3. How many bit strings of length seven contain exactly five 1s?

4. How many bit strings of length seven contain at least five 1s?
of ways to choose the 5 positions in the string where to place the 1's (and on the other spots to place the 0's)
 $C(7,5)$ (or $C(7,2)$)

$$C(7,5) + C(7,6) + C(7,7) \quad (\text{addition rule})$$

5. How many bit strings of length seven either begin with two 0s or end with three 1s?

$$2^5 + 2^4 - 2^2 \quad (\text{principle of inclusion and exclusion})$$

6. How many permutations of the letters ABCDEF are there?

$$6! = P(6,6)$$

7. How many permutations of the letters ABCDEF contain the string DAC?

$$4! = P(4,4) \quad (\text{treat DAC as a single character})$$

8. A department contains 8 men and 10 women. How many ways are there to form a committee with three members if the committee should contain at least one woman and at least one men? (The positions in the committee are identical, so the order of the members does not matter.)

$$C(8,2) \cdot C(10,1) + C(8,1) \cdot C(10,2) \quad \text{or} \quad C(16,3) - C(8,3) - C(10,3)$$

Committee with 2 men & 1 woman | 1 woman & 2 men | ~~or~~ complement rule

9. What is the coefficient of x^6y^4 in $(x+y)^{10}$?

$$C(10,4) = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} = 210$$

Note that the answer $C(8,1) \cdot C(10,1) \cdot C(16,1)$ is incorrect as double counting occurs: e.g. the committee Mark, Anna, John is counted again as John, Anna, Mark.

Name: Solution Key

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Quiz 5 - v.B

MAD 2104

Summer A 2015

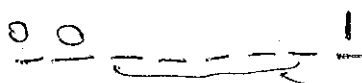
Each of the following problems is worth 3 pts. It's OK if you just give the answers, but if the answer is wrong, I can give you some partial credit only if you provide some justification which gets close to the solution of the problem (justification can be short - e.g. complement rule, addition rule, etc.)

1. How many bit strings of length eight are there?

$$2^8$$

2. How many bit strings of length eight start with two 0s and end with a 1?

$$2^5$$



five free bits

3. How many bit strings of length eight contain exactly six 0s?

$$C(8, 6) \text{ (or } C(8, 2))$$

4. How many bit strings of length eight contain at least six 0s?

$$C(8, 6) + C(8, 7) + C(8, 8)$$

5. How many bit strings of length eight either begin with three 1s or end with two 0s?

$$2^5 + 2^6 - 2^3$$

6. How many permutations of the letters $ABCDE$ are there?

$$P(5, 5) = 5!$$

7. How many permutations of the letters $ABCDE$ contain the string CA ?

$$4! = P(4, 4)$$

8. A department contains 10 men and 8 women. How many ways are there to form a committee with three members if the committee should contain at least one woman and at least one men? (The positions in the committee are identical, so the order of the members does not matter.)

See solution in recitation.

9. What is the coefficient of x^3y^7 in $(x + y)^{10}$?

$$C(10, 7) = C(10, 3) = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$$