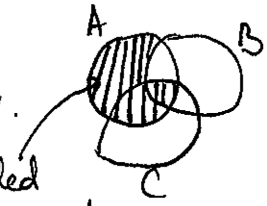
Name: Solution Key PanthID: ______________

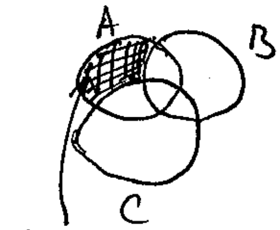
Group work 05/20 MAD 2104 Summer A 2015

1. (5 pts; 2effort + 3correctness) In each case, prove or disprove. To disprove, it is enough to give a counterexample. (Note that while a Venn diagram helps, it is not a substitute to a proof or a concrete counterexample.)

(*a*) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$, for all sets A, B, C.



Shaded set represents $A \setminus (B \setminus C)$



$(A \setminus B) \setminus C$

Venn diagram suggests that in general $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$

To show this, is enough to give one example: Let $A = \{a, b\}$, $B = \{b, c\}$, $C = \{a, c\}$.

Then $A \setminus (B \setminus C) = \{a, b\} \setminus \{b\} = \{a\}$. $(A \setminus B) \setminus C = \{a\} \setminus \{a, c\} = \emptyset$

Thus, $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$.

Exercise: (a) Show that the inclusion $(A \setminus B) \setminus C \subseteq A \setminus (B \setminus C)$ is always true

(b) Show that $[(A \setminus B) \setminus C] \cup (A \cap C) = A \setminus (B \setminus C)$

(*b*) $(A \cap B) \times C = (A \times C) \cap (B \times C)$, for all sets A, B, C.

This is always true. Proof:

$(x, y) \in (A \cap B) \times C \iff x \in A \cap B \wedge y \in C \iff$

$\iff (x \in A \wedge x \in B) \wedge y \in C \iff x \in A \wedge x \in B \wedge y \in C \wedge y \in C \iff$

$\iff (x \in A \wedge y \in C) \wedge (x \in B \wedge y \in C) \iff$

$\iff (x, y) \in A \times C \wedge (x, y) \in B \times C \iff (x, y) \in (A \times C) \cap (B \times C)$

2. (5 pts; 2effort + 3correctness) Consider the set $A = \{a, b, c, d\}$. In each case, give an example of a relation $\mathcal{R}$ on A satisfying the conditions. Just give the example, no further justification is needed.

(a) $\mathcal{R}$ is symmetric, anti-symmetric, but not reflexive.

One of several possible examples

$R = \{(a, a), (b, b)\}$ symmetric, anti-symmetric but not reflexive since $(c, c) \notin R$

Even $R = \emptyset$ would have worked

(b) $\mathcal{R}$ is reflexive, transitive, but not symmetric and not anti-symmetric.

One of several possible examples

$R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, d)\}$

R is reflexive since $\{(a, a), (b, b), (c, c), (d, d)\} \subseteq R$

R is transitive as you can check

R is not symmetric since $(c, d) \in R$ but $(d, c) \notin R$

R is not anti-symmetric since $(a, b) \in R$, $(b, a) \in R$ but $a \neq b$.