Name: _____ **Panther ID:** _____

Exam 1 **MAP 2302: Summer B 2018**

1. (10 pts) Label each 1st order differential equation below with its type - exact, separable, homogeneous (as in section 2.2B), linear, Bernoulli (and not linear), or 'none of the above'. You **DO NOT** have to solve any of them.

(a) $\dfrac{dy}{dx} + \dfrac{2y}{x} = xy^3$

(b) $(y^2+1)\,dx + (x^2+4)\,dy = 0$

(c) $(x^2+xy+y^2)\,dx - (x^2+y^2)\,dy = 0$

(d) $\dfrac{dy}{dx} + \dfrac{y}{x^2} = \cos(y)$

(e) $\dfrac{dy}{dx} + \dfrac{y}{x^2} = \cos x$

2. (15 pts) Determine the constant A so that $(Ax^2y + 2y^2)\,dx + (x^3 + 4xy)\,dy = 0$ is exact. Solve the resulting DE (implicit solution is OK).

3. (15 pts) Consider the differential equation $(\frac{dy}{dx})^2 - 4y = 0$.

(a) (6 pts) Check that the DE above has a one-parameter family of solutions of the form $y = (x + c)^2$.

(b) (5 pts) Does the DE above have a unique solution such that $y(1) = 1$? Briefly justify.

(c) (4 pts) Is part (b) in contradiction with the fundamental theorem for existence and uniqueness of 1st order IVP? Briefly justify.

4. (15 pts) Solve the initial value problem: $(x + y)\,dx - x\,dy = 0$, with $y(1) = 3$.

5. (16 pts) These are True/False questions. Answer and give a brief justification. Interpret *a good way* to mean *a standard efficient way.* (4 pts each).

(a) The equation $y'' + x^2y' + 4xy = e^x$ is a linear, 2nd order, ODE. **True** **False**

Justification:

(b) A good way to approach the DE $(x+2y+3)\,dx + (3x+6y-2)\,dy = 0$ is to set $z = x+2y$, to make it separable. **True** **False**

Justification:

(c) A good way to approach the DE $\frac{dy}{dx} + x^3y = xy^3$ is to set $y = vx$ to make it separable. **True** **False**

Justification:

(d) The IVP $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = 1$, has a unique solution defined on an interval $(1-\delta, 1+\delta)$. **True** **False**

Justification:

6. (10 pts) A stone weighing 4 lbs falls from rest towards ground from a great height. As it falls, the air resistance is numerically equal to $v/2$ (in lbs), where v is the velocity (in feet per second). Set up an IVP in v and t that would allow you to solve for $v(t)$ (but you DO NOT have to solve it). You can use standard formulas such as $F = ma$ and $g = 32$ ft/sec^2.

7. (15 pts) Solve the following linear 1st order DE

$$\frac{dy}{dx} + \frac{2y}{x} = 4x$$

8. (12 pts) Choose ONE to prove. If you have time and do both proofs, the second proof may give some (small) bonus towards a problem where you scored lower.

(A) State and prove the theorem stating that a certain substitution will change a general (non-linear) Bernoulli equation into a linear 1st order DE.

(B) Find (with proof) a condition on M, N that insures that a differential equation
$M(x, y)dx + N(x, y)dy = 0$ has an integrating factor of the type $\mu = \mu(x)$.