Name: ______________________ **Panther ID:** __________

Exam 2 **MAP 2302: Summer B 2018**

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, headphones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations will lead to a score of 0 on this exam.

1. (25 pts) These are True/False questions. Circle your answer and give a brief justification (5 pts each).

(a) If y_1 and y_2 are particular solutions of a 3rd order linear homogeneous DE with variable coefficients, then $y_1 - y_2$ is also a solution.

True **False**

Justification:

(b) The Variation of Parameters method can be applied to find a particular solution of $y'' - 2y' + y = xe^x \ln x$.

True **False**

Justification:

(c) If f_1, f_2, f_3 are all solutions for $(x^2+1)y'' + (x-1)y' + (x+3)y = 0$, then $\{f_1, f_2, f_3\}$ are linearly dependent.

True **False**

Justification:

(d) The Wronskian of $\{e^{2x}, e^{-2x}\}$ is always non-zero. **True** **False**

Justification:

(e) If a free undamped motion satisfies $x(t) = 3\sin(2t) + 4\cos(2t)$ then it oscillates with an amplitude exceeding 4.823.

True **False**

Justification:

2. (20 pts) (a) (6 pts) Find the general solution $y(t)$ of $y'' - 5y' + 4y = 0$.

(b) (8 pts) Find the general solution of $y'' - 5y' + 4y = 3e^{2t}$.

(c) (6 pts) Use the UC method to write the form for a particular solution of $y'' - 5y' + 4y = 5\sin(t) + 3te^{4t} + 10$ including constants A, B, C etc as needed. You do **NOT** have to compute the constants for this part.

3. (15 pts) Find the solution of the IVP: $9x^2y'' + 3xy' + y = 0, \quad y(1) = 3,\ y'(1) = 2$.

4. (15 pts) An 8-lb weight stretches a hanging spring 6 inches from its natural position. The weight is then pulled down another 6 inches and released at time $t = 0$. The medium offers resistance equal to $4x'$ where x' is the velocity in feet per sec. Find a formula for the displacement $x(t)$. Note that gravitational acceleration is $g = 32$ ft/sec^2.

5. (15 pts) Given that $y = e^{2x}$ is a solution of $(2x+1)y'' - 4(x+1)y' + 4y = 0$, find a linearly independent solution by reducing the order. Write the general solution.

Hint: If you forgot the procedure, take a look on Problem 6 (A) (on next page).

6. (15 pts) Choose ONE proof. If you have time to do both proofs, the second score may give some bonus towards a previous problem where your score is lower.

(A) Suppose $f(x)$ is a non-trivial solution of the second order homogeneous linear ODE

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0\ .$$

Show that the substitution $y = f(x)v$, followed by $w = v'$, will reduce the above ODE to a first order homogeneous linear ODE in w.

(B) Derive the formulas for $c_1'(x)$ and $c_2'(x)$ from the VP method.
That is, show that if y_1, y_2 are linearly independent solutions of $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$, then a particular solution for $a_2(x)y'' + a_1(x)y' + a_0(x)y = b(x)$ is given by

$$y_p(x) = c_1(x)y_1(x) + c_2(x)y_2(x)\ , \quad \text{where}$$

$$c_1'(x) = -\frac{b(x)y_2(x)}{a_2(x)w(x)}\ , \ c_2'(x) = \frac{b(x)y_1(x)}{a_2(x)w(x)} \quad \text{and } w(x) \text{ denotes the Wronskian of } y_1, y_2.$$