Name: ______________________________ **Panther ID:** _______________

Final Exam **MAP 2302: Summer B 2018**

1. (12 pts) Answer True or False. No justification is necessary (unless the question looks ambiguous). (2 pts each)

(a) The UC method can be applied to find a particular solution of $y'' + y = \sec x$ **True** **False**

(b) Every solution of $y'' + 9y = 0$ can be expressed as $y(t) = c\cos(3t + \phi)$, for some constants c and ϕ. **True** **False**

(c) Given that $y = e^{2x}$ solves $y' = 2y$, a second linearly independent solution can be found by reducing the order.

True **False**

(d) Given that $y = e^x$ is a solution of $(x^2 + x)y'' - (x^2 - 2)y' - (x + 2)y = 0$, a second linearly independent solution can be found using the substitution $y = e^x v$.

True **False**

(e) The differential equation $(x^2 + x)y'' - (x^2 - 2)y' - (x + 2)y = 0$ has no singular points. **True** **False**

(f) The IVP problem $y' = y^{1/3}$, $y(1) = 0$, has unique solution the trivial solution $y(x) = 0$. **True** **False**

2. (15 pts) For each differential equation below indicate its type (be specific) and write (only) the first step(s) towards solving it. For example, given the DE $y^{(4)} + 3y' + 2y = 0$, your answer should be: "linear homogeneous DE of order 4 with constant coefficients", and first step, "solve the characteristic equation $\lambda^4 + 3\lambda + 2 = 0$."
DO NOT spend time trying to solve completely any of the DEs in this problem. It is NOT required. (5 pts each)

(a) $x^2y'' - 3xy' + 2y = 0$

(b) $(x^2 + y^2)\,dx - 2xy\,dy = 0$

(c) $\dfrac{dx}{dt} + \dfrac{x}{t^2} = \dfrac{1}{t^2}$

3. (15 pts) Solve the I.V.P. $y' = -\frac{y}{x} + \frac{y^2}{x}$, $y(1) = 2$.

4. (15 pts) Use Laplace transform to solve the system

$$\begin{cases} x' + y = e^{2t} \\ y' + x = 0 \end{cases}$$

with initial conditions $x(0) = 3,\ y(0) = 0$.

5. (15 pts) Determine the value of the constant A such that $(Ax^2y + 2y^2)\, dx \; + \; (x^3 + 4xy)\, dy = 0$ is exact. Solve the resulting DE.

6. (15 pts) Find a series solution to the I.V.P. $y'' - 3xy = 0,\ y(0) = 1,\ y'(0) = 0$. OK to list just the first three non-zero terms, but you should also list the recursive relation.

7. (12 pts) Find the Laplace transform $L(h(t))$, where

$$h(t) = \begin{cases} 2, & 0 < t < 3 \\ 0, & 3 < t < 6 \\ 2, & t > 6 \end{cases}$$

8. (12 pts + 6 pts bonus) If $F(s) = \frac{1}{s^2(s+3)}$, find $L^{-1}\{F(s)\}$ either using partial fractions or convolution. You'll get the bonus points if you solve the problem both ways.

9. Choose ONE proof, but you could do TWO for possible bonus. Note the different values.

(A) (12 pts) Show that $L(f') = sL(f) - f(0)$ (assume that f is nice and s is large enough).

(B) (12 pts) Show that $L(u_a(t)f(t-a)) = e^{-as}F(s)$, where $F(s) = L(f(t))$.

(C) (14 pts) Solve the I.V.P. $x'' + \lambda^2 x = \delta_{t_0}$, $x(0) = x_0, x'(0) = v_0$, where x_0, v_0, λ are known constants. Describe the behaviour of the solution $x(t)$ before and after the moment $t = t_0$. What practical situation could be modelled by the I.V.P. above? Hint: $L(\delta_{t_0}) = e^{-st_0}$.

Note: The values of ALL problems on this exam is changed to 12pts each. Exceptions are problem 8 where you could still get the bonus of 6 pts and problem 9 where you could receive 14 pts for option C.