Name: _____ **Panther ID:** _____

Exam 2 MAT 3501 Fall 2018

1. (15 pts) For each of the following, answer if the statement is True or False. Then give a brief justification of your answer. (2 pts answer, 3 pts justification)

(a) $x + 1$ is a factor of $x^{2019} + 1$. **True** **False**

Justification:

(b) If a is algebraic and b is transcendental then $a + b$ is transcendental. **True** **False**

Justification:

(c) Any convergent sequence is monotone. **True** **False**

Justification:

2. (15 pts) (a) (8 pts) Prove that $\sqrt[3]{5}$ is an irrational, algebraic number.

(b) (7 pts) Prove that $(\sqrt[3]{5})^{\sqrt[3]{5}}$ is transcendental (be sure to state the theorem you are using).

3. (15 pts) (a) (8 pts) Use long division to find the quotient and the remainder when $3x^3 + 7x^2 + 9$ is divided by $x^2 - x + 1$.

(b) (7 pts) Explain how part (a) is used to find (eventual) oblique asymptotes for the graph of the rational function $f(x) = \frac{3x^3+7x^2+9}{x^2-x+1}$.

4. (18 pts) (a) (6 pts) Find the polar form of $\sqrt{3} - i$.

(b) (6 pts) Use part (a) to simplify $(\sqrt{3} - i)^{2019}$. For full credit, put your final answer in the form $a + bi$.

(b) (6 pts) Find the principle value of $(\sqrt{3} - i)^{3i}$.

5. (15 pts) Find, if possible, a polynomial $p(x)$ of degree 5, with real coefficients, which also satisfies the conditions $p(i) = p(1-i) = 0$, $p(0) = 1$, $p(1) = 2$. If such $p(x)$ does not exist explain why, if $p(x)$ exists, you could leave it in factored form, but only with real numbers in its expression.

6 and 7. (up to 30 pts) Choose TWO of the following proofs. Note the different point values.

(A) (12 pts) State and prove the Rational Root Theorem (it's OK if you give the detailed proof for just 1/2 of it).

(B) (12 pts) State and prove the quadratic formula.

(C) (18 pts) Prove that the sequence $a_n = \left(1+\frac{1}{n}\right)^n$ is monotone and bounded and, therefore, is convergent. What is the limit?