Name: _____ **Panther ID:** _____

Worksheet 10/17 MAT 3501 Fall 2019

1. You are told that the equation $2x^4 + x^3 - x^2 + x - 3 = 0$ has two rational roots. Use the rational root theorem to make a short list of candidates and find at least one of the rational roots. (Not to spend too much time with computations, I'll tell you the other provided that your short list is fine.)

(b) Now use part (a) to find all roots (real or complex) of the equation $2x^4 + x^3 - x^2 + x - 3 = 0$.

2. (a) Use the rational root theorem to show that $\sqrt[3]{5}$ is irrational.

(b) Use the rational root theorem to show that if n is not a perfect square then $\sqrt{n}$ is irrational.

3.* Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ ... \ + a_1 x + a_0$ be a polynomial with integer coefficients. Show that if a_0, a_n and $f(1)$ are odd, then $f(x) = 0$ has no rational roots.

4.* Let a, b be positive integers. Describe all possible cases for (a, b) so that the number $\sqrt{a + \sqrt{b}}$ is rational.