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**Worksheet - Nov. 26 - Geometry and Optimization**

MAT 3501

Fall 2019

1. Suppose that conical paper cups are made from disk sectors by gluing along the flat sides. Suppose we need conical paper cups that hold a given volume  $V_0$ . How should the paper cup be made in order to minimize the area of the paper used (assume there is no waste)? Show that the optimal paper cup will correspond to a certain optimal angle of the sector and find this optimal angle.

*Note:* This will give a quick way to test if an actual conical paper cup has minimal paper usage.

2. Find the dimensions of the cone of minimal volume that can be circumscribed to a sphere of a given radius  $r$ .

3. James Bond is at the southernmost point of a circular lake. He needs to get to the northernmost point (diametrically opposite). It is known that James Bond runs twice as fast as he can swim and he can maintain his top speed (both for running and for swimming) for a long time. He can swim directly across the lake, he can run all the way around the lake, or he can try a combination of swimming and running.

(a) What path should James Bond take in order to minimize the time of the trip?

(b) What path should James Bond take in order to maximize the time of the trip (but keeping the appearance that he is doing his best)?