Name: ______________________________ **Panther ID:** _______________

Worksheet - Sep. 24-26 MAT 3501 Fall 2019

1. (a) Write each of the following numbers in base 10: $\overline{1234}_5$, $\overline{110101}_2$.

(b) How do you know if a number written in base 2 is an even or odd number in base 10? How do you know if a number if a number written in base 2 corresponds to a number divisible by 8 in base 10? Generalize this observation.

(c) Given the base 10 number 323, apply the algorithm to write the number in base 7 and then in base 4.

(d) Can you give an explanation why the algorithm works? (You can start with a concrete example, e.g. 323, but ideally your explanation should capture why the algorithm works in general.)

2. (a) Show that for a number written in base 10, $\overline{a_n a_{n-1} ... a_2 a_1 a_0}$, we have

$$\overline{a_n a_{n-1} ... a_2 a_1 a_0} \equiv a_0 - a_1 + a_2 - ... + (-1)^n a_n \pmod{11} \ .$$

(b) Apply part (a) to find the remainder of 987654321 when divided by 11.

3. *Twin primes* are numbers of the form $p, p+2$, both of them prime. It is still an open problem whether there are infinitely many twin primes. You are not asked to solve this open problem, but try the following:

(a) Prove that that the sum of twin primes other than 3, 5 is always divisible by 12.

(b) *Triplet primes* are numbers of the form $p, p+2, p+4$, all three of them prime. Show that 3, 5, 7 are the only triplet primes.

4. (a) Show that the equation $ax \equiv 1 \pmod{\mathrm{n}}$ has a solution if and only if $\gcd(\mathrm{a}, \mathrm{n}) = 1$.

(b) More generally than (a), show that the equation $ax \equiv c \pmod{\mathrm{n}}$ has a solution if and only if $\gcd(\mathrm{a}, \mathrm{n}) | \mathrm{c}$.

(c) Using (a), show that if p is prime, all the non-zero elements in $\mathbf{Z}_p$ have an inverse with respect to the multiplication. (This implies that when p is prime, $(\mathbf{Z}_p, +, \cdot)$ is a (commutative) *field*.)