Name: Solution Key **Panther ID:** ____________

Exam 1 Trigonometry Summer A 2016

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Turn off your cell phone at the beginning of the exam and place it in your bag, NOT in your pocket.
3. **No calculators, of any kind,** *are allowed. Any other electronic devices, notes, texts or formula sheets are also not allowed. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.*

Problems 1 through 6 of the exam are multiple choice. Please circle the correct answer **and give a brief justification** of your answer. In each case, 3 pts for answer, 3 pts for justification.

1. Find the reference angle of 550°.

A. 360°

B. 50°

C. 10°

D. 80°

E. None of the above

Justification:

$550^\circ = 360^\circ + 190^\circ = 360^\circ + 180^\circ + 10^\circ$

so the reference angle for 550° is $\underline{10^\circ}$.

2. Convert $\frac{5\pi}{12}$ to degrees.

A. 75°

B. 150°

C. 60°

D. 50°

E. None of the above

Justification:

$\frac{5\pi}{12}$ rads $= \frac{5\times 180^\circ}{12} = 5\times 15^\circ = 75^\circ$

3. Find the length of an arc of a circle of radius 6 meters formed by a central angle of 100°.

A. 600 meters

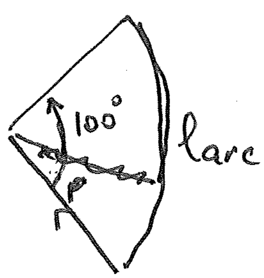
B. 20π meters

C. 36π meters

D. 100π meters

E. None of the above

Justification:



$l_{arc} = \theta \cdot r$

in radians

$l_{arc} = 10\cancel{0} \cdot \frac{\pi}{18\cancel{0}} \cdot 6 = \frac{10\pi}{3}$ meters

4. Find the exact value of

$$\cot 40° - \frac{\sin 50°}{\sin 40°}$$

A. 0

B. 1

C. -1

D. 2

E. None of the above

Justification : $\sin(50°) = \cos(90° - 50°) = \cos(40°)$

Thus $\cot(40°) - \dfrac{\sin(50°)}{\sin(40°)} =$

$= \cot(40°) - \dfrac{\cos(40°)}{\sin(40°)} = \cot(40°) - \cot(40°)$

$= 0$

5. State the domain of the function $f(x) = \sec x$.

A. All real numbers

B. [-1,1]

C. All real numbers except integer multiples of π

D. All real numbers except odd integer multiples of $\pi/2$

E. None of the above

Justification:

$f(x) = \sec x = \dfrac{1}{\cos x}$

so $f(x)$ is not defined at the points x where $\cos x = 0$

But there are all odd multiples of $\frac{\pi}{2}$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\right)$

6. Find the phase shift of the function $f(x) = 2\sin(2\pi x - 4) + 4$.

A. -4

B. 4

C. 2π

D. 2

E. None of the above

Justification:

$f(x) = 2\sin\left(2\pi\left(x - \frac{4}{2\pi}\right)\right) + 4$

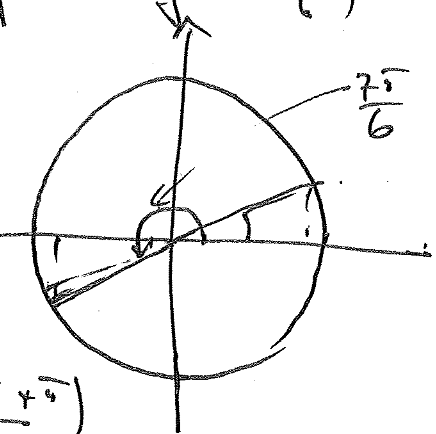
The phase-shift is

$\dfrac{4}{2\pi} = \dfrac{2}{\pi}$

7. (12 pts) Find the exact value of each of the following. Specify if the expression is undefined.

$\sin(90°) =$ 1

$\cos(7\pi/6) =$ $-\frac{\sqrt{3}}{2}$ (reference angle is $\frac{\pi}{6}$)



$\cos(\pi) =$ -1

$\cot(135°) =$ -1

(reference angle is $45°$)

$\sec(-2\pi/3) =$ $\frac{1}{\cos\left(-\frac{2\pi}{3}\right)} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$

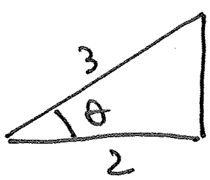
$\tan(11\pi/2) =$ $\tan\left(\frac{10\pi+\pi}{2}\right) = \tan\left(5\pi+\frac{\pi}{2}\right) = \tan\left(\frac{\pi}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)}$ undefined

8. (16 pts) (a) (8 pts) Given that θ is an acute angle and that $\cos(\theta) = \frac{2}{3}$, find the exact value of

$\sin(\theta) =$ $\frac{\sqrt{5}}{3}$

$\tan(\theta) =$ $\frac{\sqrt{5}}{2}$

$\csc(\theta) =$ $\frac{1}{\sin\theta} = \frac{3}{\sqrt{5}}$



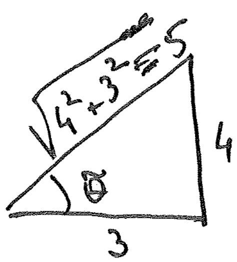
$\sqrt{3^2-2^2} = \sqrt{5}$

(b) (8 pts) Given that θ is an angle in the 3rd quadrant and that $\tan(\theta) = \frac{4}{3}$, find the exact value of

$\sin(\theta) =$ $-\frac{4}{5}$

$\cos(\theta) =$ $-\frac{3}{5}$

$\cot(\theta) =$ $\frac{1}{\tan\theta} = \frac{3}{4}$

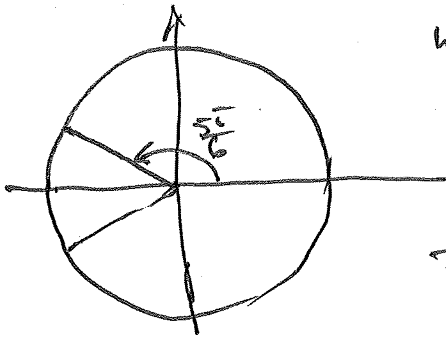


negative signs because θ is in the 3^{rd} quadrant.

In this picture, $\tilde{\theta}$ is a reference angle for θ, that is, $\tilde{\theta}$ is in the first quadrant and

$\tan\tilde{\theta} = \frac{4}{3}$

9. (8 pts) Find the value(s) of θ, $0 \le \theta \le 2\pi$, so that $\cos\theta = -\frac{\sqrt{3}}{2}$.



We know that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ and $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

Thus $\theta = \frac{5\pi}{6}$ or $\theta = \frac{7\pi}{6}$

10. (18 pts) Due to tide, the height of the water, H, in feet, at a boat deck t hours after 6 A.M. is given by

$$H(t) = 10 + 4\sin\left(\frac{\pi}{6}t\right).$$

(a) (6 pts) What is the maximum height of the water (high tide), what is the minimum height of the water (low tide)?

The maximum height $H_{max} = 10 + 4 = 14$ ft
The minimum height $H_{min} = 10 - 4 = 6$ ft
(we just used that $-1 \le \sin\theta \le 1$)

(b) (6 pts) What is the period of this function and what does this say about the tide?

The period is $\frac{2\pi}{\frac{\pi}{6}} = 12$ hours, so this says that in a 12 hour interval there is a high-tide and a low-tide.
In a day (24 hours) there are two high-tides and two low-tides.

(b) (6 pts) At what time(s) during the day does the low tide occur?

The low-tide occurs when $\sin\left(\frac{\pi}{6}t\right) = -1$, so

when $\frac{\pi}{6}t = \frac{3\pi}{2}$, so when $t = \frac{3 \cdot 6}{2} = 9$

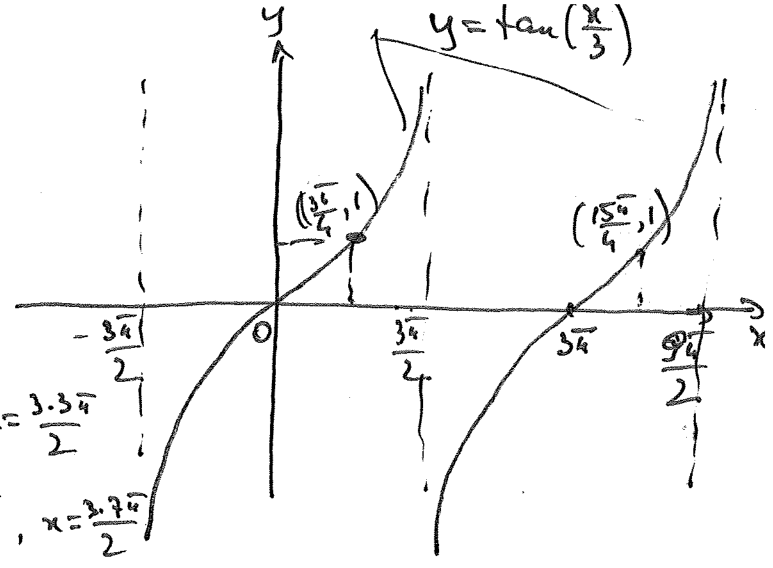
Since $t = 0$ corresponds to 6.A.M., $t = 9$ corresponds to 3P.M.
Thee next occurence of the low tide is at
$t = 9 + 12 = 21$ which corresponds to 3 A.M.
(you could find $t = 21$ also by $\frac{\pi}{6}t = \frac{3\pi}{2} + 2\pi \mid \cdot 6 \Rightarrow$
$\Rightarrow \pi t = 9\pi + 12\pi \Rightarrow \pi t = 21\pi \Rightarrow t = 21$)

11. (10 pts) Graph two cycles of $y = \tan(x/3)$.

Period is 3π

and the vertical asymptotes occur at $x = -\frac{3\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{3\cdot 3\pi}{2}$

$x = \frac{3\cdot 5\pi}{2}$, $x = \frac{3\cdot 7\pi}{2}$



12. (10 pts) Chose ONE.

(a) State and prove the Pythagorean relation between $\sec\theta$ and $\tan\theta$. (You may use without proof the Pythagorean relation between $\sin\theta$ and $\cos\theta$.)

(b) State and prove the formula for the area of a sector of a circle of radius r and of angle θ radians. (You may use without proof the formula for the area of a circle.)

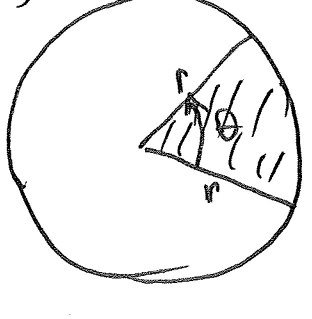
~~see notes~~

(a) $\sin^2\theta + \cos^2\theta = 1$ Divide both sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$ so

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

(b)



By proportionality

$$\frac{A_{sector}}{A_{disk}} = \frac{\theta}{2\pi}$$ ← in radians

so

$$A_{sector} = \frac{\theta}{2\pi}\cdot A_{disk} = \frac{\theta}{2\pi}\cdot \pi r^2$$

So $\boxed{A_{sector} = \frac{1}{2}\theta\cdot r^2}$

↑ angle in radians