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Exam 3 Trigonometry Summer A 2016

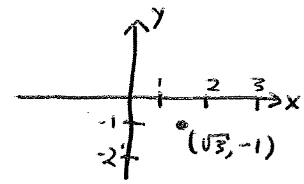
Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Turn off your cell phone at the beginning of the exam and place it in your bag, NOT in your pocket.
3. **A scientific calculator is needed for some problems. If you do not have a scientific calculator, leave your answers in calculator ready form. Graphing calculators are not allowed.**
4. Notes, texts or formula sheets are prohibited. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.

1. (8 pts) (a) On a rectangular coordinate system, plot the point given in polar coordinates by $(r = -2, \theta = 5\pi/6)$.

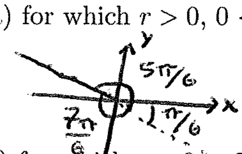
$x = r\cos(\theta) = -2\cos(5\pi/6) = -2\left(\frac{-\sqrt{3}}{2}\right) = \sqrt{3}$

$y = r\sin(\theta) = -2\sin(5\pi/6) = -2\left(\frac{1}{2}\right) = -1$



(b) Find other polar coordinates (r, θ) of the point in part (a) for which $r > 0$, $0 < \theta < 2\pi$.

$\left(r = 2, \theta = \frac{11\pi}{6}\right)$



(c) Find other polar coordinates (r, θ) of the point in part (a) for which $r > 0$, $-2\pi < \theta < 0$.

$\left(r = 2, \theta = \frac{-\pi}{6}\right)$

(d) Find other polar coordinates (r, θ) of the point in part (a) for which $r < 0$, $-2\pi < \theta < 0$.

$\left(r = -2, \theta = \frac{-7\pi}{6}\right)$

2. (6 pts) Rectangular coordinates (x, y) are given. In each case, find polar coordinates (r, θ) for the point.

(a) $A(-2\sqrt{3}, 2)$ ← 2^{nd} quadrant.

$x = -2\sqrt{3}$.
$y = 2$.
So, $r^2 = x^2 + y^2$
$= 4 \cdot 3 + 4$
$= 16$

$\Rightarrow \boxed{r = 4}$ or $\boxed{r = -4}$.

$\tan(\theta) = \frac{y}{x} = \frac{2}{-2\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\Rightarrow \theta = \arctan\left(\frac{-1}{\sqrt{3}}\right)$

$\Rightarrow \cancel{\boxed{\theta = \frac{5\pi}{6}}}$ or $\boxed{\theta = \frac{11\pi}{6}}$

A valid answer could be $(r=4, \theta=5\pi/6)$ or $(r=-4, \theta=\frac{11\pi}{6})$

[illegible] and [illegible] are not in the co-domain of arctan, so $\arctan\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$. Therefore, another valid answer is $\left(r=-4, \theta=\frac{-\pi}{6}\right)$.

(b) $B(-3, -3)$ $x = -3$, $y = -3$, [redacted] 3rd quadrant.

So, $r^2 = x^2 + y^2 = 9 + 9 = 18$

$\Rightarrow \boxed{r = \sqrt{18}}$ or $\boxed{r = -\sqrt{18}}$.

$\tan(\theta) = \frac{y}{x} = \frac{-3}{-3} = 1$

$\Rightarrow \theta = \arctan(1)$

$\Rightarrow \boxed{\theta = \frac{\pi}{4}}$, but other valid values for θ include $\frac{5\pi}{4}$ and $-\frac{3\pi}{4}$.

A valid answer could then be $\left(r = -\sqrt{18}, \theta = \frac{\pi}{4}\right)$.

3. (30 pts) Solve each of the following triangles. Specify if no solution, or more than one solution exist.

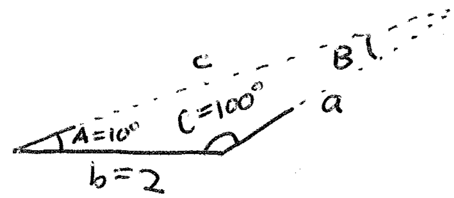
(a) Solve the triangle with angles $A = 10^o$, $C = 100^o$, and side $b = 2$.

Only one solution for the constraints.

$A + C + B = 180°$

$\Rightarrow 110° + B = 180°$

$\Rightarrow \boxed{B = 70°}$



By law of Sines, $\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$

$\Rightarrow \frac{2}{\sin(70°)} = \frac{a}{\sin(10°)}$

$\Rightarrow \boxed{a = \frac{\sin(10°)\cdot 2}{\sin(70°)}}$

Also, $\frac{c}{\sin(C)} = \frac{b}{\sin(B)}$

$\Rightarrow \frac{c}{\sin(100°)} = \frac{2}{\sin(70°)}$

$\Rightarrow \boxed{c = \frac{\sin(100°)\cdot 2}{\sin(70°)}}$

(b) Solve the triangle with $a = 8$, $b = 9$, and angle $A = 60^o$.

By law of Sines, $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \Rightarrow \frac{8}{\sin(60)} = \frac{9}{\sin(B)}$

$\Rightarrow \sin(B) = \frac{9}{8}\cdot\sin(60°)$

$\Rightarrow \sin(B) = \frac{9\sqrt{3}}{16}$

$\Rightarrow \boxed{B = \arcsin\left(\frac{9\sqrt{3}}{16}\right)°} \approx 77°$

but we should also check

$B = 180° - 77° \approx 103°$

(since $\sin(180° - 77°) = \sin(77°)$)

~~Therefore, $C = (180 - 60 - \arcsin(\frac{9}{16}))°$~~

Both solutions are possible:

Triangle 1: $A = 60°$, $B = 77°$, $C = 180° - 60° - 77° = 43°$

$\frac{c}{\sin(43°)} = \frac{8}{\sin(60°)} \Rightarrow c = \frac{8\sin(43°)}{\sin(60°)} \approx 6.3$

$a = 8$, $b = 9$, $c \approx 6.3$

and

Triangle 2 (for (b))

$A = 60°$, $B = 103°$, $C = 180° - 60° - 103° = 17°$

$\frac{c}{\sin(17°)} = \frac{8}{\sin(60°)} \Rightarrow c = \frac{8\sin(17°)}{\sin(60°)} \approx 2.7$

So $a = 8$, $b = 9$, $c \approx 2.7$

(c) Solve the triangle with $a = 4$, $b = 4$, and $c = 4\sqrt{3}$.

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

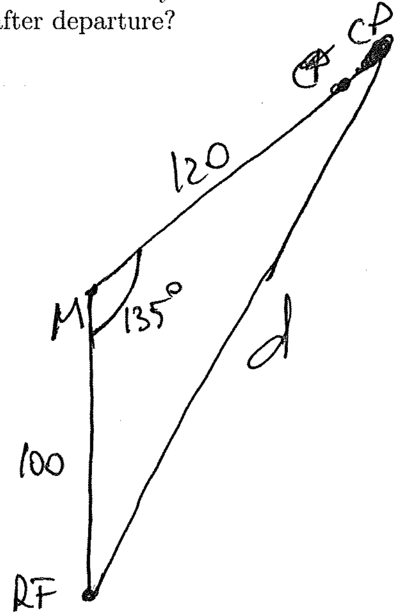
$3\cdot 4^2 = 4^2 + 4^2 - 2\cdot 4^2 \cos C$

$3 = 2 - 2\cos C \Rightarrow \cos C = -\frac{1}{2} \Rightarrow C = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3} = 120°$

Since $a = b \Rightarrow \Delta$ is isosceles so $A = B$ (can be seen from Law of Sines too)

So $A = B = \frac{180° - C}{2} = \frac{60°}{2} = 30°$ and $C = 120°$

4. (10 pts) Two cruise-ships, the Carnivore Princess and the Royal Flounder, are leaving the port of Miami at the same initial time. The Carnivore Princess sails exactly NorthEast at a constant speed of 30 km/hour. The Royal Flounder sails exactly South at a constant speed of 25 km/hour. What is the distance between the two ships 4 hours after departure?



Use the Law of Cosines

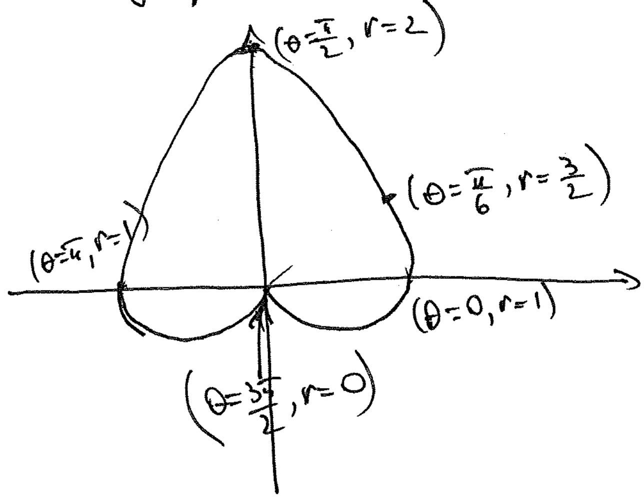
$$d^2 = 100^2 + 120^2 - 2\cdot 100\cdot 120\cos(135^\circ)$$

$$d^2 = 10000 + 14400 + \cancel{2}\cdot 12000\left(\frac{\sqrt{2}}{\cancel{2}}\right)$$

$$d = \sqrt{24400 + 12000\sqrt{2}} \approx 203.4 \text{ km}$$

5. (10 pts) Sketch the graph of $r = 1 + \sin(\theta)$. The polar coordinates of at least five points should be given.

The graph is a cardoid symmetrical w.r.t. y-axis



6. (24 pts) Solve each of the following equations on the interval $[0, 2\pi)$.

(a) (8 pts) $3(1-\sin x) = \cos^2 x$. *Hint:* Use the appropriate identity first.

$$3(1-\sin x) = 1-\sin^2 x$$

Easiest way to do it next is probably to factor the right side

$$3(1-\sin x) = (1-\sin x)(1+\sin x)$$

so $0 = (1-\sin x)(1+\sin x) - 3(1-\sin x) \iff$

$\iff 0 = (1-\sin x)[1+\sin x - 3] \iff 0 = (1-\sin x)(\sin x - 2)$

so $\sin x = 1$ or $\sin x = 2 \leftarrow$ not possible

$\longrightarrow$ so only solution is $\boxed{x = \frac{\pi}{2}}$

(b) (8 pts) $\cot(3x) = \sqrt{3}$. *Hint:* Be sure to find **all** solutions in $[0, 2\pi)$.

$\checkmark$ $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$ so

$$3x = \frac{\pi}{6} + k\pi$$

so $x = \frac{\pi}{18} + \frac{k\pi}{3}$ $k \in \mathbb{Z}$

Solutions in $[0, 2\pi)$ are

$x_0 = \frac{\pi}{18}$ $(k=0)$ $x_1 = \frac{\pi}{18} + \frac{\pi}{3} = \frac{7\pi}{18}$ $x_2 = \frac{13\pi}{18}$ $x_3 = \frac{19\pi}{18}$ $x_4 = \frac{25\pi}{18}$, $x_5 = \frac{31\pi}{18}$

so there are 6 solutions in $[0, 2\pi)$

(c) (8 pts) $\cos(2x) + 5\cos x + 3 = 0$. *Hint:* Use the appropriate double angle formula first.

Use $\cos(2x) = 2\cos^2 x - 1$

so the equation becomes

$2\cos^2 x - 1 + 5\cos x + 3 = 0$ or $2\cos^2 x + 5\cos x + 2 = 0$

This factors $(2\cos x + 1)(\cos x + 2) = 0$

so $\cos x = -\frac{1}{2}$ or $\cos x = -2 \longrightarrow$ not possible

so $\boxed{x_1 = \frac{2\pi}{3}}$ or $\boxed{x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}}$

7. (10 pts) Transform the polar equation $r = 2\cos\theta - 6\sin\theta$ in rectangular coordinates. Then, complete the squares to show that the equation represents a circle and graph it.

$r = 2\cos\theta - 6\sin\theta \mid \cdot r \Rightarrow r^2 = 2r\cos\theta - 6r\sin\theta$

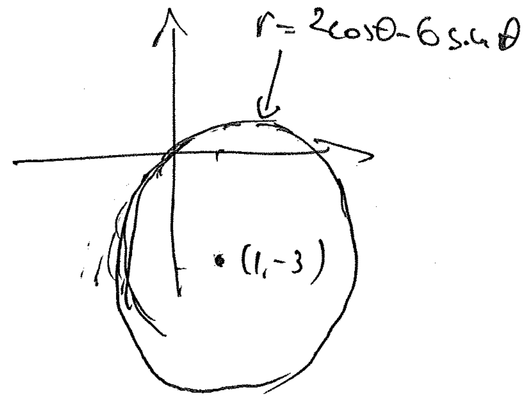
$\Rightarrow x^2 + y^2 = 2x - 6y \Rightarrow$

$\Rightarrow x^2 - 2x + y^2 + 6y = 0 \Rightarrow$

$\Rightarrow x^2 - 2x + 1 + y^2 + 6y + 9 = 1 + 9$

$(x-1)^2 + (y+3)^2 = 10$

So, it is a circle with center at $(1,-3)$ and radius $r = \sqrt{10}$



8. Choose ONE. Only one will be graded. Note the different point values.

(A) (12 pts) State and prove the Law of Cosines.

(B) (8 pts) State and prove the Law of Sines.

See notes or text