Name: ______________________________ **Panther ID:** _______________

Worksheet June 10 Trigonometry Summer A 2016

1. In each part, you are given the polar coordinates of a point. First plot the point, and then find the rectangular coordinates of each point. Finally, give one different polar coordinates representation of the same point.

(a) $(r = 3, \theta = 90^\circ)$ (b) $(r = -5, \theta = \frac{\pi}{6})$ (c) $(r = 2, \theta = \frac{5\pi}{4})$

2. Convert each rectangular equation to a polar equation that expresses r in terms of θ.

(a) $x^2+y^2 = 25$ (b) $(x+3)^2+y^2 = 9$ (c) $x^2 = 3y$

3. Convert each polar equation to a rectangular coordinate equation.

(a) $r\sin\theta = -3$ (b) $r = 2\cos\theta$ (c) $r^2\sin(2\theta) = 6$

4. Convert to rectangular coordinates to show that the graph of $r = a\cos\theta$ is a circle with center at $(a/2, 0)$ and radius $a/2$.

For polar graph paper see the link on http://mathstat.fiu.edu/useful-information/math-resources/trigonometry/

5. (a) Suppose that in a triangle we know two sides, a, b, and the angle C between them. Show that the area of the triangle is given by $A = \frac{ab\sin C}{2}$.

(b) Consider a regular pentagon inscribed in a circle of radius r and let A_5 denote the area of this pentagon. Find a formula for A_5 in terms of the radius r of the circle. For now, do not evaluate the trig. function that appears in your answer, but express in radians the angle.
Hint: Using the center of the circle, divide the pentagon into 5 congruent triangles.

(c) With the same reasoning, if a regular decagon is inscribed in a circle of radius r, find a formula for the area A_{10} of the decagon in terms of r.

(d) Generalize the formulas in parts (b) and (c). If a regular polygon with n-sides is inscribed in a circle with radius r, find a formula for the area A_n of the n-gon in terms of r and n.

(e) Now use your calculator to evaluate the trig. function in your answer and get (approximative) decimal expressions for A_5, A_{10}, A_{100}. (For example, for the area of a regular octogon, you'd get $A_8 \approx 2.83r^2$).

(f) In words, what do you think you are finding from A_n as n is getting larger and larger?

Note: With the notion of *limit*, which you'll learn in Calculus, the process you did above will become completely rigorous. What you discovered numerically above is the following limit

$$\lim_{k \to +\infty} k \sin\left(\frac{\pi}{k}\right) = \pi$$

6. This problem is a homework due Wednesday, June 15. This is a repeat of problem 5, but this time for perimeter instead of area.

(a) Consider a regular pentagon inscribed in a circle of radius r and let P_5 denote the perimeter of this pentagon. Find a formula for P_5 in terms of the radius r of the circle. For now, do not evaluate the trig. function that appears in your answer, but express in radians the angle.
Hint: Just as before, using the center of the circle, divide the pentagon into 5 congruent triangles and use the law of cosines in one of these triangles.

(c) With the same reasoning, if a regular decagon is inscribed in a circle of radius r, find a formula for the perimeter P_{10} of the decagon in terms of r.

(d) Generalize the formulas in parts (b) and (c). If a regular polygon with n-sides is inscribed in a circle with radius r, find a formula for the perimeter P_n of the n-gon in terms of r and n.

(e) Now use your calculator to evaluate the trig. function in your answer and get (approximative) decimal expressions for P_5, P_{10}, P_{100}. (For example, for the area of a regular octogon, you'd get $P_8 \approx 6.123r$).

(f) In words, what do you think you are finding from P_n as n is getting larger and larger?