Interest Rate Futures

Chapter 6

Day Count Convention

- The day count convention defines:
  - The period of time to which the interest rate applies.
  - The period of time used to calculate accrued interest (relevant when the instrument is bought or sold).

Day Count Conventions in the United States

Treasury Bonds:	Actual/Actual
T-bills and Money Market Instruments:	Actual/360
Corporate and Municipal Bonds:	30/360

Example 1

- Treasury Bond: 8% Actual/ Actual in period.
  - With Treasury bonds paying semi-annually, half the coupon rate (8%/2=4%) is earned between coupon payment dates.
  - Accrued interest is on an Actual basis.
  - When coupons are paid on March 1st 2013 and Sept 1st 2013, how much interest is earned between March 1st 2013 and April 1st 2013 ?

- Answer:
  - Assume the principal is $100. There are 184 actual days in the March 1st – September 1st reference period.
  - There are 31 actual days between March 1st and April 1st 2013.
  - The interest accrued/earned is thus: $4 x (31/184) = $0.67

Example 2

- Treasury-Bill: 8%, Actual/360.
  - 8% is earned in 360 days.
  - Accrued interest is calculated by dividing the actual number of days in the period by 360.
  - This means, by the way, that the interest earned in an entire year is 365/360 times the quoted rate.
  - How much interest is earned between March 1st and April 18th 2013, assuming a $100 face value?

- Answer
  - There are 48 actual days in the March 1st – April 18th period.
  - The interest accrued/earned is therefore:

$100 x 0.08 x (48/360) = $1.067

Example 3

- Corporate/Municipal Bond: 8%, 30/360.
  - With corporate/muni bonds paying semi-annually, half the coupon rate (8%/2=4%) is earned between coupon payment dates.
  - Assumes 30 days per month and 360 days per year.
  - When coupons are paid on March 1st and Sept 1st 2013, how much interest is earned between March 1st and May 21st 2013?

- Answer
  - Assume the principal is $100.
  - There are 180 days in the March 1st – September 1st reference period.
  - There are 30+30+20=80 days between March 1st and May 21st 2013 (Think from morning to morning, with 30 days per month that is in full).
  - The interest accrued/earned is therefore:

$$\$4 \times (80/180) = \$1.78$$

A February Interesting Example

- How many days of interest are earned between February 28th, 2014 and March 1st, 2014 when
  - day count is Actual/Actual in period? (Treasury bonds)
  - day count is 30/360? (Corporate/Municipal bonds)

- Answer:
  - From "morning to morning", there is only one actual day: the day of February 28th.
  - With a 30-day month count, there are 3 days: Feb 28th, Feb 29th, Feb 30th (even though the last two do not exist!)

Treasury Bill Prices in the US

- Treasury Bills and money market instruments are often quoted using a *discount rate*: the interest earned *as a percent of the final value (principal)* rather than of the initial price paid for the instrument.
- For example, if the price of a 91-day T-bill is quoted as 8 (percent), the interest earned is 8% of the face value ($100) per 360 days.
- Over the 91-day life span of the T-bill, one earns $100x0.08x91/360 = $2.0222 and so the cash price of the T-bill is 100 – 2.0222 = 97.9778
- Therefore the true rate of interest is 2.0222 / 97.9778 = 2.064%.
- In general, we have: $P = \frac{360}{n}(100 - Y)$

  where Y is cash price per \$100, here \$97.9778

  and where P is quoted price as a percent, here 8.

Treasury Bond Price Quotes

- In the US, Treasury bonds are quoted in dollars and 32nds of a dollar.
- The quoted price is for a bond with a face value of $100.
- Example: a quote of 90-05 indicates that if the bond has a face value of $100,000 its price will be (90+5/32)x1,000 = $90,156.25
- This *quoted price* is also known as the *clean price*.
- The actual *cash price* that has to be paid by the purchaser of the bond is also known as the *dirty price*.

Cash price = Quoted price + Accrued Interest

Bond prices in the US are usually quoted using the “clean” price: why?

- The reason is that the cash or dirty price (the true value of the bond) will decrease sharply by the amount of the coupon on the day that the coupon is paid (like a stock price after a dividend payment).

- To avoid this saw-tooth pattern, the clean price gradually subtracts the accumulated/accrued *upcoming* interest/coupon payment such that, the closer the payment, the more has been deducted already.

- By the time the payment occurs, by definition the full payment has accrued and there is therefore no “shock” to the clean price.

- This makes it easier to isolate the effect of a change in credit rating or in the yield curve. It separates the effect of the coupon from profits/losses.

Treasury Bond Price Quotes in the U.S: Example

- Assume it is March 5th 2013 and the bond is an 11%-coupon bond maturing on July 10th 2028, with a quoted price of 95-16.
- Government bonds pay coupons semiannually and the latest coupon was paid on January 10th 2013. The next coupon date is July 10th 2013.
- The actual number of days between Jan 10 and Mar 5 is 54.
- The actual number of days between Jan 10 and Jul 10 is 181.
- Each coupon pays $100 x 0.11 / 2 = $5.50 (on Jan 10 and Jul 10)
- The accrued interest on Mar 5 is: $5.50 x 54 / 181 = $1.64
- The cash price per $100 face value is thus $95.50+$1.64 = $97.14
- The cash price of a $100,000 face value bond is thus: $97,140

Treasury Bond Futures

- Popular interest rate futures contracts are: Treasury bond futures, 10-year Treasury note futures, 5-year Treasury note futures, 2-year Treasury note futures, 30-Day Fed Funds Rate futures, and Eurodollar futures.
- But the bonds actually delivered by the party on the short side of the futures contract are not necessarily of those exact same maturities.

- Acceptable government bonds/notes to deliver are:
  - For Ultra-Treasury bond futures: remaining maturity > 25 years
  - For “regular” Treasury bond futures: 15 years < remaining maturity < 25 years
  - For 10-year Treasury note futures: 6.5 years < remaining maturity < 10 years
  - For 5-year Treasury note futures: remaining maturity ≈ 5 years
  - For 2-year Treasury note futures: remaining maturity ≈ 2 years

Treasury Bond Futures

- Treasury bond futures contracts are quoted in dollars and 32nd of a dollar, like in the Treasury bond market.

  Ex: 151-20 = 151+20/32 = 151.625

  (note that you might also see the quote expressed as: 151-20/32)

- 10-year Treasury note futures contracts are quoted in dollars and *to the nearest half of a 32nd* of a dollar.

  Ex: 134-215 = 134+21.5/32 = 134.671875

  (note that you might also see the quote expressed as: 134-43/64)

- 5-year and 2-year Treasury note futures contracts are quoted in dollars and *to the nearest quarter of a 32nd* of a dollar.

  Ex: 124-152 = 124+15.25/32 = 124.4765625

  (note that you might also see the quote expressed as: 124-61/128)

Treasury Bond Futures

- The Treasury bond futures contract allows the party with the short position to choose to deliver any government bond with a maturity left between 15 and 25 years.

The cash price received by the party with short position =

$$\text{Most Recent Settlement Price} \times \text{Conversion factor} + \text{Accrued interest}$$

Example

- Most recent settlement price = 90-00
- Conversion factor of bond delivered = 1.3800
- Accrued interest on bond = $3.00 per $100 of face value

- The price received by the party on the short side of the futures contract for the bond delivered to the long side is:

$$1.3800 \times 90.00 + 3.00 = \$127.20$$

(per $100 of principal)

- The actual price received would be $127,200.

Conversion Factor

- The conversion factor for a bond is approximately equal to the value of the bond (per dollar of principal) on the assumption that the yield curve is flat at 6% with semiannual compounding.

- The bond maturity and the times to the coupon payment dates are rounded down to the nearest 3 months for the purposes of the calculation (in order to be able to produce comprehensive tables).

- After rounding, if the bond lasts for an exact number of six-month periods, the first coupon is assumed to be made in 6 months.

- If it does not (i.e. there are extra 3 months), the first coupon is assumed to be paid after 3 months, and accrued interest is subtracted (for the 3 months preceding the present).

Conversion Factor: Example #1

- Assume a 10% coupon bond with 20 years and 2 months to maturity.
- Rounding down to the nearest 3 months, the bond is thus assumed to have exactly 20 years to maturity.
- Since 20 years is an exact number of 6-month periods, the first coupon payment is therefore assumed to be made after 6 months.
- Assuming a $100 face value, and a flat 6% per annum discount rate with semiannual compounding, the value of the bond is:

$$\sum_{i=1}^{40} \frac{100(0.10/2)}{(1+0.06/2)^i} + \frac{100}{(1+0.06/2)^{40}} = \$146.23$$

- The conversion factor is therefore 1.4623

Conversion Factor: intuition

- The reason for the 6% discount rate used in the computation of the conversion factor is the fact that *bond futures prices inherently assume a 6% coupon rate*.
- Therefore, if the bond being delivered *actually has a 6% coupon rate*, the conversion factor will be equal to 1 since it will discount future $6 coupon payments (well, $3 twice a year equating $6 per year) at a rate of 6%.
- Therefore the short party would actually receive the futures settlement price without any modification (except for possible accrued interest).
- But if the bond has a *higher* coupon payment, the short party should be compensated for the fact that he his giving a higher-paying bond. The conversion factor will reflect that since it will be higher than 1.
- Conversely, if the bond has a *lower* coupon payment, the short party should receive a little less because of the fact that he is giving a lower-paying bond. The conversion factor will reflect that since it will be lower than 1.

Conversion Factor: Example #2

- Assume an 8% coupon bond with 18 years and 4 months to maturity.
- Rounding down to the nearest 3 months, the bond is thus assumed to have exactly 18 years and 3 months to maturity.

- We first compute the price of the bond at a "time point" 3 months from now when the first coupon happens. We will then discount the bond price by 3 months to "bring it back to the present". Finally we will have to subtract the accrued interest for the prior 3 months.
- Assuming a \$100 face value, and a flat 6% per annum discount rate with semiannual compounding, the value of the bond is:

$$100(0.08)/2+\sum_{i=1}^{36}\frac{100(0.08/2)}{(1+0.06/2)^i}+\frac{100}{(1+0.06/2)^{36}}=\$125.83$$

- To "bring back" the price to the present we need to discount by 3 months. If the 6-month rate is 6%/2=3%, a 3-month rate is (1+0.03)^(1/2) - 1 = 1.4889%.
- At "time 0", the price becomes \$125.83/(1+0.014889) = \$123.99 = *cash price* of the bond (since what we just computed is the true/cash/actual value of the bond).
- To get the *quoted price* of the bond, one must subtract the accrued interest from the prior 3 months. The accrued interest is half a coupon payment, thus \$4/2 = \$2.
- The quoted price is therefore \$123.99 – 2 = \$121.99 and the conversion factor 1.2199

Cheapest-to-Deliver Bond

- The party with the short position, when delivering the bond, receives:

  Most recent settlement price x Conversion factor + Accrued interest

- The cost of purchasing the bond is:

  Quoted bond price + Accrued interest

- Thus the cheapest-to-deliver bond is the one minimizing:

Quoted bond price - settlement price x Conversion factor

CBOT T-Bonds & T-Notes Futures prices

Exact theoretical futures prices are difficult to determine because there are many factors that affect the futures price:

- Delivery can be made any time during the delivery month
- Any of a range of eligible bonds can be delivered
- The wild card play

Eurodollar Futures

- The 3-month Eurodollar futures contract is the most popular interest rate futures contract in the US.
- A Eurodollar is a dollar deposited in a bank outside the United States (can be a US or foreign bank).
- A 3-month Eurodollar futures contract is a futures contract on the interest that will be paid (by someone who borrows at the Eurodollar interest or deposit rate, same as 3-month LIBOR rate) on $1 million for a future period of 3 months.
- Maturities are in March, June, September, and December, for up to 10 years in the future.
- One contract is on the rate earned on $1 million
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25

Eurodollar Futures (continued)

- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month) the final settlement price is:

100 – R

where R is the actual 3-month Eurodollar interest rate on that day.

- R is expressed with quarterly compounding.
- R uses an actual/360 day count convention.

Eurodollar Futures (continued)

- If, at expiration, R=0.5%, the final settlement price is 99.500
- The size of these contracts is such that a one basis point (0.01) move in the futures quote corresponds to a gain or loss of $25 per contract.

- This is because if R is the Eurodollar rate, and the loan is for 3 months (1/4 of a year), and the loan principal is $1 million, the interest paid is: 1,000,000 x R x ¼.
- And therefore, given a one basis point change in the rate R (0.01% or 0.0001), the change in the interest paid in dollars is: $1,000,000 x 0.0001 x 0.25 = $25.

- A one basis point (0.01) move in the futures quote also corresponds to a one basis point (0.01%) change in the futures interest rate, albeit in the opposite direction.

Eurodollar Futures (continued)

- When a Eurodollar futures quote increases by one basis point, a trader who is long one contract gains $25 and a trader who is short one contract loses $25.
- When a Eurodollar futures quote decreases by one basis point, a trader who is long one contract loses $25 and a trader who is short one contract gains $25.
- For example, if the settlement price changes from 99.120 to 99.230, a trader with a long position gains 11x25 = $275 per contract.
- A trader with a short position loses $275 per contract.

Example

- An investor wants to lock in the interest rate for a 3-month period starting Sep 16th 2015, for $100 million (to invest).
- The Sep 2015 Euro futures quote is 96.500, meaning that the investor can lock in a rate of 100-96.5 = 3.5% per annum.
- The investor hedges by *buying* 100 contracts.
- Suppose that on Sep 16th 2015, the 3-month Eurodollar rate is 2.6%. The final settlement in the contract is thus at a price of 97.400
- What are the gains/losses on the futures contract and on the subsequent investment at the Eurodollar rate prevailing on Sep 16th ? Show that 3.5% is indeed the interest received.

Example

- The difference in basis points is 100 x (97.40-96.50) = 90
- The investor gains: 100 x 25 x 90 = 225,000 on the Eurodollar futures contract.
- The interest earned on the 3-month investment is:

  100,000,000 x 0.25 x 0.026 = 650,000

- Once we add the futures gains to the interest, the total earned is:

  650,000 + 225,000 = 875,000

- Checking that 3.5% was indeed locked in:
- 100,000,000 x 0.25 x 0.035 = 875,000
- The hedge worked !

Formula for Contract Value

- If Q is the quoted price of a Eurodollar futures contract, the value of one contract is $10{,}000 \times [100-0.25(100-Q)]$
- For example, if the futures price is 99.570, the contract value is:
  10,000 x [100 – 0.25x(100-99.570)]
- So the contract value is $998,925.
- Note that for a given interest rate move, both the quoted price and the contract value move together (in a direction opposite to that of the rate).
- Also note that gains and losses computed from changes in the contract value are consistent with gains/losses computed from the quote as we did before, with a $25 gain or loss per change in basis point.

Contract Value: Example/Comparison

- If Q is the *quoted price* of a Eurodollar futures contract, the CME defines the *contract price* as: $10{,}000 \times [100-0.25(100-Q)]$

- If the quote Q went from 99.570 to 99.500, under the "old way" of calculating gains and losses we have:
- A change of 99.500 - 99.570 = -0.07 or a loss of 7 basis points.
- Therefore we have a loss of $25 x 7 = **$175**.

- Correspondingly, the Eurodollar futures contract price went from
- 10,000 x [100-0.25(100-99.570)] to 10,000 x [100-0.25(100-99.500)]
- in other words the contract value went from $998,925 to $998,750
- Therefore we have a change in value of $998,750 - $998,925 = -**$175**.

Forward Rates and Eurodollar Futures

- Eurodollar futures contracts last as long as 10 years (in time to maturity).
- For Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate.
- Two reasons for that:
- Futures are settled daily whereas forwards are settled once.
- Futures are settled at the beginning of the underlying three-month period; Forward Rate Agreement is settled at the end of the underlying 3-month period.

Forward Rates and Eurodollar Futures (continued)

- A "convexity adjustment" often made is:

  $$\text{Forward Rate} = \text{Futures Rate} - 0.5\sigma^2 T_1 T_2$$

  - T_1 is the start of period covered by the forward/futures rate.
  - T_2 is the end of period covered by the forward/futures rate (90 days later that T_1).
  - σ is the standard deviation of the change in the short rate per year.

Duration

- A bond B providing cash flows c_i at times t_i is worth $B = \sum_{i=1}^{n} c_i e^{-yt_i}$

- The duration of a bond that provides cash flow c_i at time t_i is defined as

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

  where B is its price and y is its yield (continuously compounded)

- Differentiating B with respect to y gives us

$$\frac{\Delta B}{\Delta y} = -\sum_{i=1}^{n} c_i t_i e^{-yt_i} \quad \text{or} \quad \Delta B = -\Delta y \sum_{i=1}^{n} c_i t_i e^{-yt_i} \quad \text{or} \quad \Delta B = -\Delta y B \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

- And therefore: $\frac{\Delta B}{B} = -D \Delta y$

Duration (Continued)

- Very generally, when the yield y is expressed with compounding m times per year, we have:

$$\Delta B = -\frac{BD\Delta y}{1 + y/m}$$

- And the expression $\dfrac{D}{1 + y/m}$

  is referred to as the “modified duration”

Duration Matching

- Duration matching involves hedging against interest rate risk by matching the durations of assets and liabilities.
- It provides protection against small parallel shifts in the zero curve.

Duration-Based Hedge Ratio

V_F Contract Price for Interest Rate Futures

D_F Duration of Asset Underlying Futures at Maturity

P Value of portfolio being Hedged

D_P Duration of Portfolio at Hedge Maturity

Duration-Based Hedge Ratio

- If the yield *y* changes by a small amount (and by the same amount for all maturities), it is approximately true that: $\Delta P = -PD_P\Delta y$

- And if one has N* futures contracts, the total change in value of these N* futures contracts is: $\Delta V_F = -V_F D_F \Delta y N^*$

- Therefore, the number N* of futures contracts needed to hedge against an uncertain Δy is given by:

$$N^* = \frac{PD_P}{V_F D_F}$$

Example

- A 3-month hedge is required for a $10 million portfolio. Duration of the portfolio in 3 months will be 6.8 years.
- The December T-bond futures price is 93-02 so that contract price is $93,062.50 (since a T-bond futures is for the delivery of $100,000 worth of face value of Treasury bonds).
- The duration of the expected cheapest-to-deliver bond at contract maturity is 9.2 years.
- The number of contracts to *short* for a 3-month hedge is:

$$N^* = \frac{10,000,000 \times 6.8}{93,062.50 \times 9.2} = 79.42$$

Example

- Assume that between August 2nd and November 2nd interest rates fall and the value of the bond portfolio increases from $10 million to $10,450,000.
- On November 2nd, The Treasury bond futures price is 98-16. This corresponds to a contract price of $98,500.00
- There is a loss of 79 x (98,500.00-93,062.50) = $429,562.50 on the Treasury bond futures contracts.
- The portfolio position thus only changes by:

  $450,000.00 - $429,562.50 = $20,437.50

GAP Management

This is a more sophisticated approach used by banks to hedge interest rate. It involves:

- “Bucketing” the zero curve.
- Hedging exposure to situation where rates corresponding to one bucket change and all other rates stay the same.

Floating-rate loans and Eurodollar futures contracts

- In April, a company borrows $15 million for 3 months (May, June, and July).

- The interest rate for each one-month period will be: LIBOR+1% (per annum).

- When the loan is negotiated, LIBOR=8% per annum.
- The LIBOR rate is quoted with monthly compounding.

- The interest for the first month (May) is therefore

  $15m x (8%+1%)/12 = $112,500 (known for certain).

Floating-rate loans and Eurodollar futures contracts

- The interest paid at the end of the second month is determined by the one-month LIBOR rate at the beginning of the second month.
- The firm takes a *short* position in the June Eurodollar futures contract, quoted at 91.88 (afraid LIBOR will go *up*).
- The contract price is: 10,000 x [100-0.25x(100-91.88)] and therefore is $979,700.
- The duration of the asset underlying the contract is 3 months (=0.25 years) since it is the 3-month Eurodollar rate.
- The duration of the liability being hedged is 1 month, which means it is 0.08333 years.

Floating-rate loans and Eurodollar futures contracts

- The number of contracts to be shorted therefore is:

  15,000,000 x 0.08333 / (979,700 x 0.25) = 5.10
- Thus 5 contracts are being shorted.

- On May 29, the 1-month LIBOR rate proves to be 8.8%, and the June futures price proves to be 91.12, indicating a contract price of 10,000 x [100-0.25x(100-91.12)] = $977,800
- The firm gains 5 x (979,700-977,800) = $9,500 on the 5 June contracts.
- The LIBOR went up from 8% to 8.8% per annum, so a 0.8% increase per annum, or a 0.8%/12 per month, applied to the $15 million principal.
- On $15 million, this extra interest translates into $10,000 for one month.
- The extra interest needed to be paid is reduced by the $9,500 gain on the futures contracts. The net extra interest is only $500.

Floating-rate loans and Eurodollar futures contracts

- Since Eurodollar futures maturities are in March, June, September, and December:
  - To hedge the third and last month (July), one must use the September contract.
  - The concept is otherwise the same.