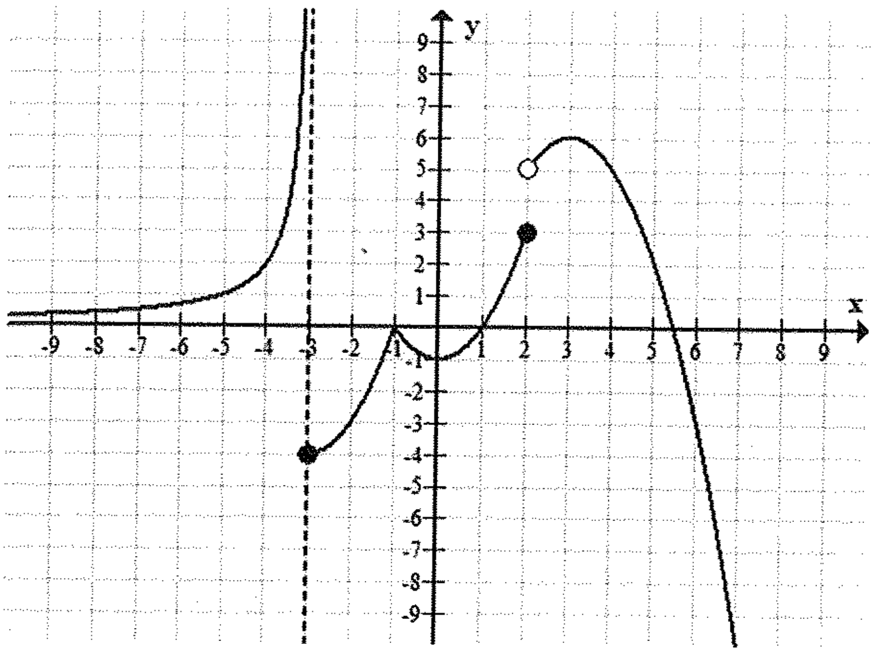
**Name:** ______________________________ **PID:** ______________________________

**Summer 2017 -- MAC 2311- Exam 1**

There are 6 problems for a total of 103 points. **Show your work**; an answer alone, even correct, may get no credit. An illegible solution will not be graded. **Calculators are not allowed.**

**Problem 1.** The graph of a function $f$ is given below. Use the graph to answer the questions that follow.




(i) (7 pts) Find the following limits (you don't have to show any work here)

$\lim_{x\to -3^-} f(x) = \infty$ $\lim_{x\to -3^+} f(x) = -4$ $\lim_{x\to -3} f(x) =$ DNE

$\lim_{x\to 2} f(x) =$ DNE $\lim_{x\to 0} f(x) = -1$

$\lim_{x\to -\infty} f(x) = 0$ $\lim_{x\to +\infty} f(x) = -\infty$

(ii) (2 pts) Specify the domain of the function $f$. $\mathbb{R}$, i.e. $(-\infty,\infty)$

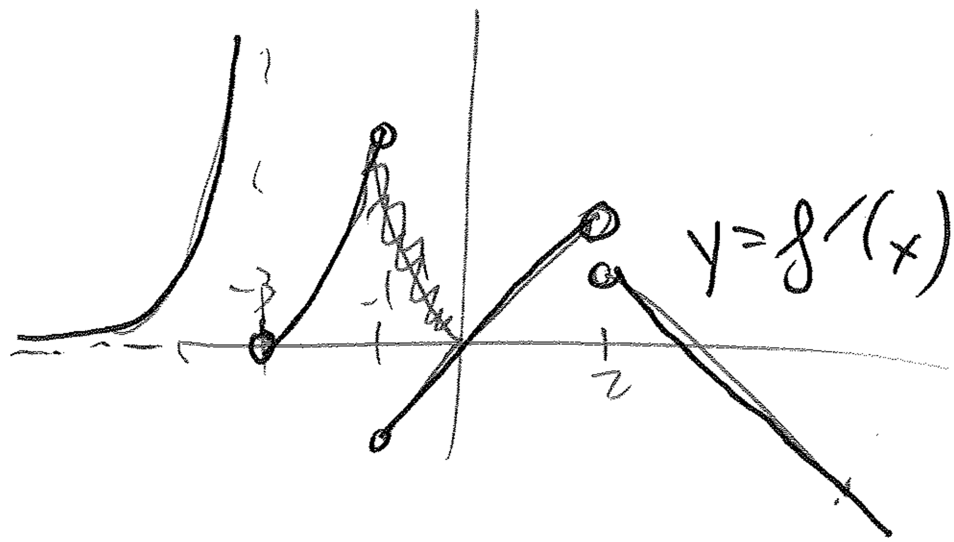
(iii) (3 pts) Is $f$ continuous everywhere? If not, give $x$ value(s) at which $f$ has a discontinuity. Specify if any of the discontinuities is removable.

disco. at $x=-3$, $x=2$.
not removable

(iv) (2 pts) Identify any point(s) $x$, where the function is continuous, but it is not differentiable. Specify if there is no such point $x$.

$x=-1$

(v) (3pts) For the function f(x) given in problem 1, sketch the graph of f'(x).




**Problem 2.** Find the following limits. Show all work and explain clearly (5 pts each).

a) $\lim_{x\to\infty} \frac{\sin 2x}{x}$ =

$\frac{-1}{x} < \frac{\sin 2x}{x} < \frac{1}{x} \Rightarrow$ 3

0

2

b) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-3x+2} = \frac{0}{0}$ = $\lim_{x\to 2} \frac{(x+3)(x-2)}{(x-1)(x-2)} = \lim_{x\to 2} \frac{x+3}{x-1}$

$= 5$

1 2 2

c) Let f be a piecewise defined function with f(x)= cos (x) for x≤0, and f(x)=3x-9 for x>0. Compute $\lim_{x \to 0} f(x)$.

DNE

(3)

$$\lim_{x \to 0^-} \cos x = \cos(0) = 1 \quad (1)$$

$$\lim_{x \to 0^+} 3x-9 = -9 \quad (1)$$

d) $\lim_{x \to +\infty} \cos(x) =$ DNE by oscillatio

e) $\lim_{x \to 0} \frac{x \tan(3x)}{\sin^2(5x)} =$

$$= \frac{x}{\sin(5x)} \quad \frac{\sin 3x}{x} \quad \frac{x}{\sin 5x} \quad \frac{1}{\cos 3x}$$

$$= \frac{1}{5} \cdot 3 \cdot \frac{1}{5} \quad \frac{1}{1} = \frac{3}{25}$$

f) $\lim_{t \to -\infty} \frac{\sqrt{2t^2 - t + 1}}{5t} = \lim_{t \to -\infty} \frac{\sqrt{2t^2}}{5t}$ 1

$= \lim_{t \to -\infty} \frac{\sqrt{2}\,|t|}{5t}$ 1

$= \lim_{t \to \infty} \frac{\sqrt{2}}{5} \cdot \frac{-t}{t}$ 3

$= -\frac{\sqrt{2}}{5}$

g) $\lim_{x \to 0} \frac{1}{x}$ DNE 3

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$ 1

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ 1

h) $\lim_{x \to 2^-} \frac{x}{2-x} = \frac{2^-}{0^+} = \infty$

0 4

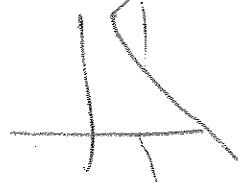
**Problem 3.** These are true or false questions. Answer (1pt) and give brief justification (2pts). Graph can serve as a justification.

(a) The graph of a function can never cross its horizontal asymptote. **True** **False**

**Justification:**

(b) The graph of a function can never cross its vertical asymptote. **True** **False**

**Justification:**



~~fails v.l.t.~~

fails vertical line test.

(c) If a function $f$ is continuous at $x=0$, then f is differentiable at $x=0$. **True** **False**

$y=|x|$

**Justification:**

not diff. at $x=0$, but cont. at $x=0$.

(d) If a function satisfies $|f(x) -5| \le 7|x-3|$ for all real numbers $x$, then $\lim_{x\to 3} f(x) = 5$. **True** **False**

**Justification:** $x \to 0 \Rightarrow f(x) \to 5$.

**Problem 4.** (10 pts) Use the limit definition of the derivative to compute $f'(x)$ for $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad (1)$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \quad (2)$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \quad (3)$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \quad (2)$$

$$= \cancel{0} \quad \frac{1}{2\sqrt{x}} \quad (2).$$

**Problem 5.** Compute f'(x) for the following (4 points each)

a) $f(x) = x\cos(x)$,

$\cos x \; \cancel{+} - x\sin x$

prod. rule: 2
details 2

b) $f(x) = \frac{x-4}{x+7}$

$$\frac{(x+7)-(x-4)}{(x+7)^2} = \frac{11}{(x+7)^2}$$

c) $f(x) = \pi^2 - 4\sqrt[5]{x} + 3x - 5/x$

$$f' = -\frac{4}{5}x^{-4/5} + 3 - \frac{5}{x^2}$$

**Problem 6.** (a) (2pts) Write the limit definition of continuity for a function *f(x)* at *x=a*.

$$\lim_{x \to a} f(x) = f(a)$$

(b) (5pts) Use this definition to determine whether or not the following function is continuous at *x=0*.

$$f(x) = \begin{cases} \dfrac{x^2+3}{x^2+1} & ,if\ x \le 0 \\ \dfrac{\sin(3x)}{x} & ,if\ x > 0 \end{cases}$$

1 $\lim_{x \to 0^-} \frac{x^2+3}{x^2+1} = 3$

2 $\lim_{x \to 0^+} \frac{\sin(3x)}{x} = 3$

$\Rightarrow \lim_{x \to 0} f(x) = 3$

2 $f(0) = 3$

3 $3 = 3$

(a) (5pts) List all asymptotes, vertical or horizontal (if any), of the function *f(x)* from part (b). Justify your answer with limits.

2 $f$ continuous everywhere $\Rightarrow$ no v.a.

3 $\lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to +\infty} f(x) = 0$

$\Rightarrow y = 0, 1$ are h.a.