Name: ______________________ Panther ID: Key

Exam 2 - MAC2311 -

Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. State whether each of the following is true or false. The questions that need an explanation are worth 3 points, those that don't are worth 1 point.

a) (3 point) If $p(x)$ is a degree n polynomial, then $p'(x)$ is a degree $n-1$ polynomial.

True $p = ax^n + bx^{n-1} + \dots \Rightarrow p' = nax^{n-1} + b(n-1)x^{n-2} + \dots$

b) (3 points) $(e^{-x})' = e^{-x}$

False: chain rule say $(e^{-x})' = e^{-x} \cdot (-1)$

c) (1 points) $(\sec^{-1}(x))' = \frac{1}{|x|\sqrt{x^2-1}}$ true

d) (3 points) If $h(x) = \tan(g(x))$, then

$$h'(x) = \sec^2(g(x)) + \tan(g'(x)).$$

False: chain rule says $\sec^2(g(x)) \cdot g'(x)$

e) (3 points) If $h(x) = \ln(g(x))$, then $h'(x) = g'(x)/g(x)$.

true by chain rule.

f) (3 points) The slope of the line tangent to $y = \ln(x)$ at $(a, \ln(a))$ approaches $+\infty$ as a approaches 0 from the right.

true: slope of tangent line $= f'(x) = \frac{1}{x}$ and $\lim_{x \to 0^+} \frac{1}{x} = \infty$.

g) (1 point) $\tan^{-1}(x) = \frac{1}{\tan(x)}$.

False.

1. Compute the following derivatives:

a)(3 points) $\frac{d}{dx}(\sin^{-1}(x))$

$$\frac{1}{\sqrt{1-x^2}}$$ (library)

b)(6 points) $\frac{d}{dx}(\tan^{-1}(x/2))$

$$\frac{1}{1+(\frac{x}{2})^2} \cdot \frac{1}{2}$$ (Chain rule)

c) (6 points) $\frac{d}{dx}(5^{\sec(x)})$ $= \ln 5 \cdot 5^{\sec x} \cdot (\sec x \tan x)$

Chain rule + library

d) (10 points) $\frac{d}{dx}(xe^{\sqrt{x+1}})$ $= e^{\sqrt{x+1}} + x \cdot \frac{1}{2\sqrt{x+1}} e^{\sqrt{x+1}}$

product + chain rule.

e)(10 points) $\frac{d}{dx}(x^{(\cos(x))})$ log. diff.

$$f(x) = x^{\cos y}$$

$$\ln f(x) = \cos x \ln x$$

$$\frac{f'(x)}{f(x)} = -\sin x \ln x + \cos x \frac{1}{x}$$

$$f'(x) = \left(-\sin x \ln \cancel{xx} + \cos(x) \frac{1}{x}\right) x^{\cos y}$$

f)(3 points) $\frac{d}{dx}(\ln(x))$ $\frac{1}{x}$ library

g) (2 points) $\frac{d}{dt}(\ln(x))$ $\frac{1}{x} \frac{dx}{dt}$ Chain rule

h) (1 points) $\frac{d}{dy}(\ln(x))$ $\frac{1}{x} \frac{dx}{dy}$ Chain rule.

2. (8) Suppose a particle moves along the curve $x^2 + y^2 + xy = 3$. If $\frac{dx}{dt} = -3$ at the point $(x, y) = (1, 1)$, what is $\frac{dy}{dt}$?

$$\frac{d}{dt}: \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + \frac{dx}{dt}y + x\frac{dy}{dt} = 0$$

$$2(-3) + 2\frac{dy}{dt} - 3 + \frac{dy}{dt} = 0$$

$$3\frac{dy}{dt} = 9$$

$$\frac{dy}{dt} = 3$$

3. (8 points) Compute $\frac{dy}{dx}$ at $(x, y) = (-1, 2)$ if $x^2 + y^2 - xy = 7$.

$$\frac{d}{dx}: \quad 2x + 2y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$

plugin $(-1,2)$:

$$-2 + 4\frac{dy}{dx} - 2 + \frac{dy}{dx} = 0$$

$$5\frac{dy}{dx} = 4$$

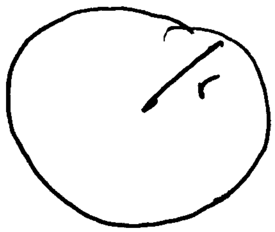
$$\frac{dy}{dx} = \frac{4}{5}.$$

4. (14 points) A stone is dropped into the middle of a pond, causing a circular wave. If the radius of this circle is growing at a rate of 3 meters per second, what will be the rate of growth of the area enclosed by the wave after 8 seconds?

Want: $\frac{dA}{dt}$

$A = \pi r^2$

Got: $\frac{dr}{dt} = 3$



$$\Downarrow$$

$$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$$

8 seconds $\Rightarrow r = 3 \cdot 8$
$= 24$

$$= \pi\, 2r \cdot 3$$
$$= \pi \cdot 2 \cdot 24 \cdot 3 = 144\pi$$

5. (3 points) Determine whether the following function, whose domain is $(-\infty, \infty)$, is invertible: $f(x) = x^3 + 9x - 5$. Justify your answer.

$f'(x) = 3x^2 + 9 > 0 \Rightarrow$ increasing
$\Rightarrow$ invertible

6. a) (4) Present the linearization of $f(x) = (1+x)^{100}$ for $a = 0$.

$f'(x) = 100 \cdot (1+x)^{99} \Rightarrow f'(a) = 100$

$L(x) = f(a) + f'(a)(x-a) = 1 + 100 \cdot x = 1 + 100x$

b) (3) Use the linearization in part a to approximate $(.99)^{100}$.

$(.99)^{100} = (1-.01)^{100} = f(-.01)$

$\approx L(-.01) = 1 + 100(-.01)$
$= \boxed{0}$

Name: ______________________ Panther ID: ____________

Exam 3 - MAC2311 -

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1: (15 points) ________

2: (20 points) ________

3: (8 points) ________

4: (16 points) ________

5: (19 points) ________

6: (7 points) ________

7: (15 points) ________

110

Total: ________/ 100

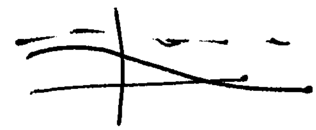
1) State whether each of the following is true or false **and give a brief explanation of your answer**

a) (3 points) If $f'(5) = 0$ and $f''(5) < 0$ then $f(x)$ has an inflection point at $x = 5$.

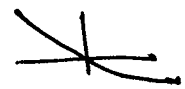
False: rel. min.

b) (3 points) If $f'(x) < 0$ for all $x \in (-\infty, +\infty)$, then $f(x)$ has no absolute maximum on $(-\infty, +\infty)$.

true: graph can be like: ~~every point is like~~



or



c) (3 points) For any functions f and g, $\int f(x)g(x)\,dx = \left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right)$.

False: $f(x) = 1$, $g(x) = 1$, then

$\int fg\,dx = \int 1\,dx = x + c$, $\int f\,dx = \int g(x) = x + C$.

so $\int f\,dx \int g(x)\,dx = (x+C)(x+C)$.

(d) (3 points) If $f(x)$ is continuous on $[-1, 2]$, then f must have an absolute minimum on this interval.

yes by Theorem.

(e) (3 points) If $\lim_{x\to\infty} f(x) = 3$, then the graph of $f(x)$ has at least one horizontal asymptote.

Yes by definitn.

2) Compute the following integrals:

(a) (4 points) $\int \left(7e^x + \frac{1}{1+x^2}\right) dx$

$7e^x + \tan^{-1}x + C$

2 2

(b) (6 points) $\int \frac{x^3-2x+\sqrt{x}-3}{x}\, dx$ $= \int x^2 - 2 + x^{-1/2} - \frac{3}{x}\, dx$ 2

$= \frac{x^3}{3} - 2x + 2x^{1/2} - 3\ln|x| + C.$ 4

(c) (4 points) $\int \sqrt{3x-7}\, dx$

$u = 3x-7$
$du = 3dx$ 2

$= \frac{1}{3}\int \sqrt{u}\, du$

$= \frac{1}{3}\frac{u^{3/2}}{3/2} + C = \frac{2}{9}(3x-7)^{3/2} + C$

2

(d) (4 points) $\int \cos(-3x)\, dx$

$u = -3x$
$du = -3\, dx$ 2

$= -\frac{1}{3}\int \cos u\, du$

$= -\frac{1}{3}\sin u + C = -\frac{1}{3}\sin(-3x) + C.$ 2

(e) (5 points) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\sin^{-1}(u) + C = \frac{1}{2}\sin^{-1}(x^2) + C$

3) (8 points) Find the absolute maximum and absolute minimum for the function $f(x) = x + \frac{1}{x}$ on the interval $(0, \infty)$. Justify your answer with calculus.

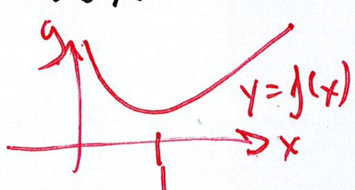
2 $\quad f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow 1 = \frac{1}{x^2} \Rightarrow x = \pm 1$

$\Rightarrow x = 1$

4 $\quad f''(x) = 2x^{-3} \Rightarrow f''(1) > 0 \Rightarrow$ rel. min.

$\Rightarrow$ abs. min.

$f(1) = \boxed{2}$



2 $\quad \lim_{x\to\infty} f(x) = \infty \Rightarrow$ not abs. max.

4) (5 points) A particle moving along the x-axis undergoes acceleration of $5\sin(t)$ meters per second, where t is time in seconds. If the initial position is $x = 0$ and initial velocity is zero, what is the position at time $t = 6$?

1 $\quad x''(t) = 5\sin t$

$x'(t) = -5\cos t + C$

2 $\quad x'(0) = 0$, $\; x'(0) = -5 + C \Big\} \Rightarrow C = 5$

$x'(t) = -5\cos t + 5$

$x(t) = -5\sin t + 5t + C$

2 $\quad x(0) = C$, $\; x(0) = 0 \Big\} \Rightarrow C = 0$

$\Rightarrow x(t) = -5\sin t + 5t$

$x(6) = \boxed{-5\sin(6) + 30.}$

5) For the function $f(x) = \frac{1}{x^2+6x}$, $= \frac{1}{x(x+6)}$

a) (4 points) Find any asymptotes. Justify your answers with limits.

$\lim_{x\to\infty} f = \lim_{x\to-\infty} f = 0 \Rightarrow y=0$ h.a. (2)

$\lim_{x\to-6^-} = \infty$ $\qquad \lim_{x\to 0^-} = -\infty$

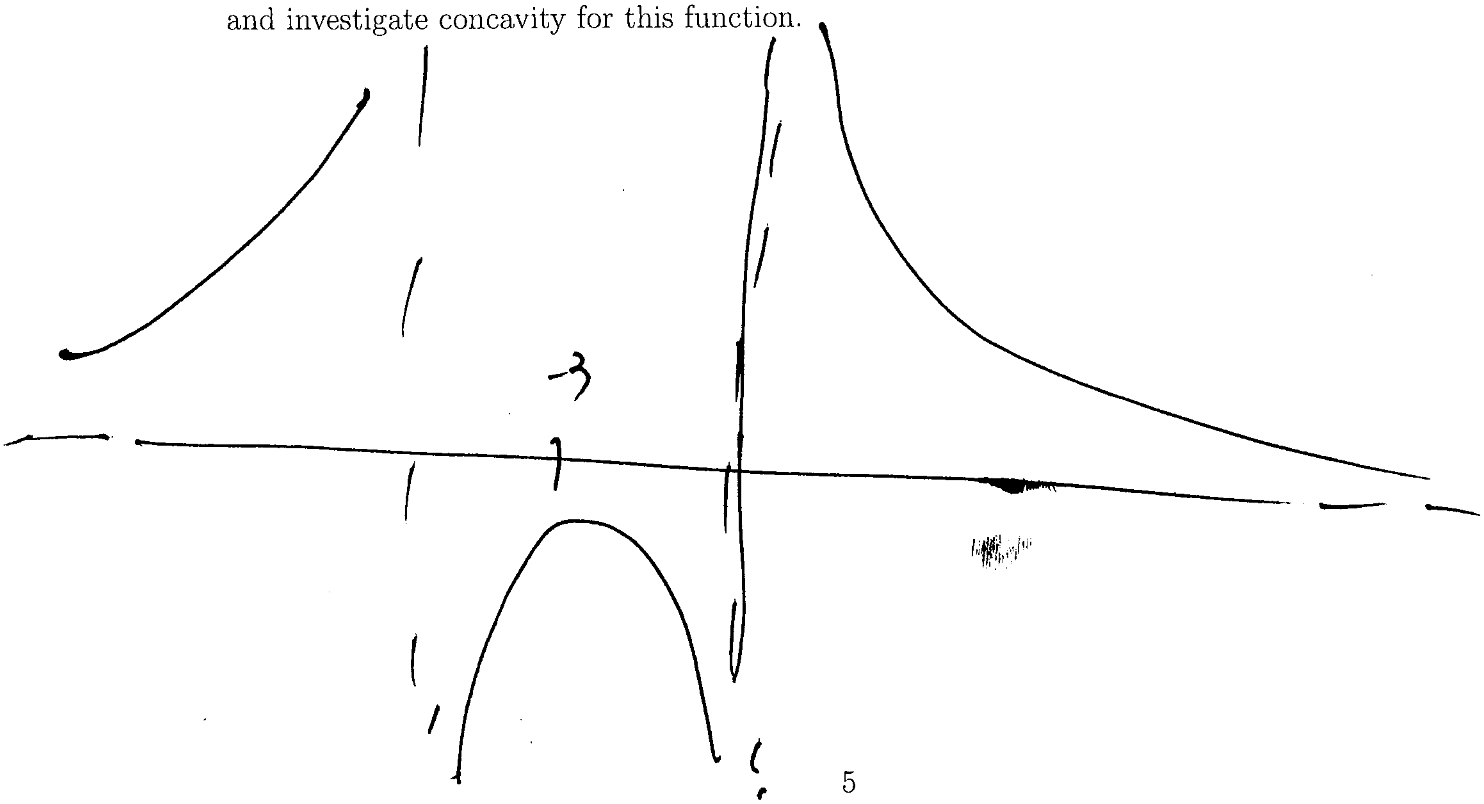
$\lim_{x\to-6^+} = -\infty$ $\qquad \lim_{x\to 0^+} = +\infty$ (3)

b) (7 points) Find the intervals on which $f(x)$ is increasing and on which it is decreasing.

$$f'(x) = -\frac{2x+6}{(x^2+6x)^2}$$

x	$-\infty$	-6	-3	0	∞
f'	+	+	−	−	
f	↗	↗	↘	↘	

c) (5 points) Sketch the graph of f, based on the calculations in a,b. Clearly mark any asymptotes and relative extrema. You are **not required** to compute the second derivative and investigate concavity for this function.



6) For the function $f(x) = \ln(x^2+1)$,

a) (4 points) Find any asymptotes. Show your reasoning both for vertical and horizontal asymptotes.

2 v.a: none because f cont.

2 h.a.. $\lim_{x \to \pm\infty} \ln(x^2+1) \text{ "=" } \ln(\infty) = \infty \Rightarrow$ no h.a.

b) (5 points) Find the intervals on which $f(x)$ is increasing and on which it is decreasing.

$$f'(x) = \frac{2x}{x^2+1}$$ 1

x	$-\infty$		0		∞
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

4

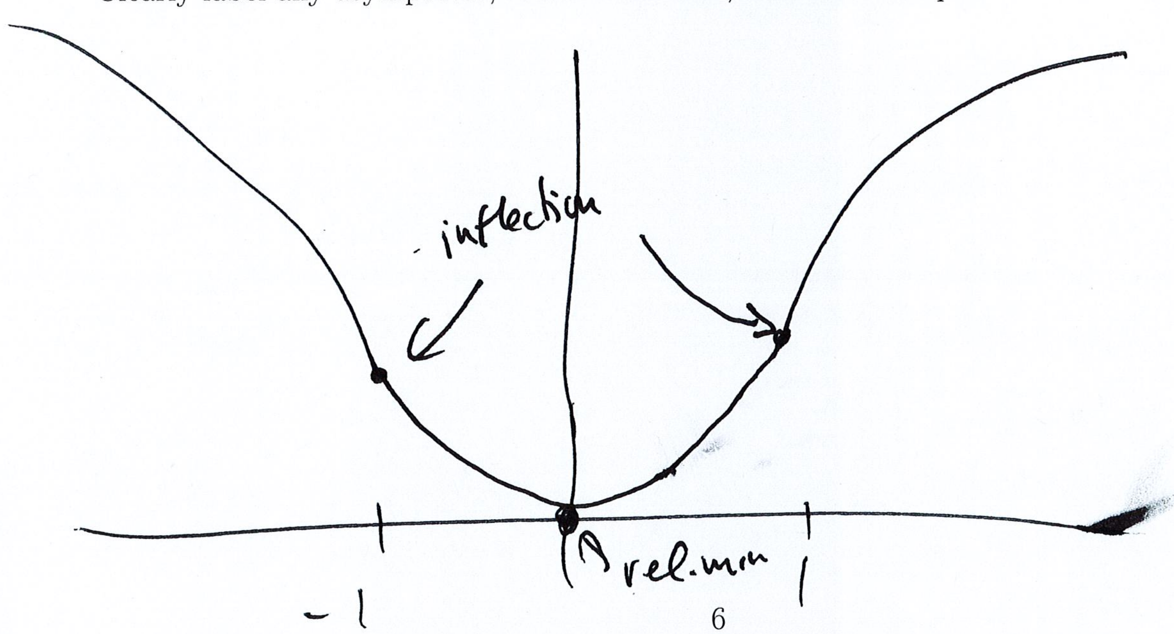
c) (5 points) Find the intervals of upward, downward concavity.

$$f''(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x)(1+x)}{(x^2+1)^2}.$$ 1

x	$-\infty$		-1		1		∞
f''		$-$		$+$		$-$	
f		$\frown$		$\smile$		$\frown$	

4

d) (6 points) Noting $f(0) = 0$, sketch the graph of $y = \ln(x^2+1)$ based on parts a, b, c. Clearly label any asymptotes, relative extrema, and inflection points.



7) Compute:
a)(3pts) $\lim_{x\to\infty} \frac{x}{\ln(x)}$

$$= \lim_{x\to\infty} \frac{1}{1/x} \quad 1$$

$$= \infty \quad 2$$

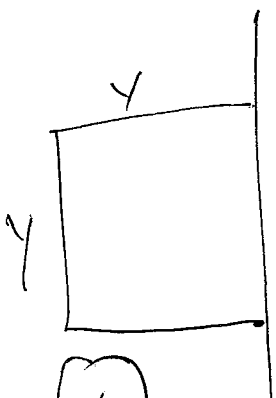
b) (5pts) $\lim_{x\to\infty}(1+\frac{1}{x^2})^{7x}$

$$\ln y = 7x \ln\left(1+\frac{1}{x^2}\right) \quad .5$$

$$3.5 \quad \lim_{x\to\infty} \frac{\ln(1+1/x^2)}{1/7x} = \lim_{x\to\infty} \frac{\frac{1}{1+1/x^2}\cdot -2x^{-3}}{\frac{1}{7}x^{-2}} = 0$$

$$1 \quad e^0 = 1$$

8) ~~(15 points)~~ A rectangular field will be bounded on one side by a river, and on the other three sides by a chain linked fence. If the field must enclosed 1000 square meters, what dimensions will minimize the amount of fencing needed?



1

$$xy = 1000 \quad 2$$

$$P = 2x + y \;(2) = 2x + \frac{1000}{x} \quad 2$$

$$(2) \quad P' = 2 - \frac{1000}{x^2} = 0 \quad 1$$

$$x^2 = 500 \Rightarrow x = \sqrt{500} = 10\sqrt{5} \quad 2$$

$$P'' = \frac{2000}{x^3} > 0 \Rightarrow \text{rel. min} \Rightarrow \text{abs. min.} \quad 2$$

$$x = \sqrt{500}$$

$$y = \frac{1000}{x} = \frac{1000}{\sqrt{500}} = 2\sqrt{500}$$