Ch. 12: (Sec 12.1)/12.3 STA-3164
Multiple regression and General linear
Model

In multiple regression we consider
relationship of response variable $y$ (dependent)
variable to several independent or explanatory
variables $x_1, x_2 \ldots x_k$ and also allows
these variables to be quantitative.

A model with $k$ independent or explanatory
variables is described as

$$ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon $$

For example, with $k=3$ independent variables
is

$$ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon $$

In general for a given data set $(x_1, x_2, \ldots, x_k)$

$\hat{y} = E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k$

$x_i$'s can be powers of other variables
like $x_2 = x_1^2$ or can be cross products
of other variables like $x_4 = x_1 x_2$ etc.
Assumptions:
\[ E(e_i) = 0 \]
\[ \text{Var}(e_i) = \sigma^2 \]
\[ e_i \text{'s are independent.} \]
\[ e_i \text{'s are normally distributed.} \]

The simple type of multiple regression, also called first order model, is a model with no powers or cross products of other variables in the model. In that case we can attach some meanings to coefficients \( \beta_1, \beta_2, \beta_3, \ldots \)

These coefficients are called partial regression coefficients.

In the case of simple model

\( \beta_i \) represent the change in \( y \) value when \( x_i \) changes by 1 unit when other \( x_j \)'s \( j \neq i \) remain the same.

We are going to analyse this model in Sec 2.3 for \( k \) quantitative variables (independent) using a sample of \( n \) measurements \((y_i, x_{i1}, x_{i2}, x_{i3}, \ldots, x_{ik})\).
to estimate $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$

by the method of least squares

i.e. $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_k$ are chosen

such that

\[ \text{sum of squares of errors} = \sum (Y_i - \hat{Y}_i)^2 \]

is minimised

where

\[ \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \ldots + \hat{\beta}_k x_{ik} \]

This process provides solution $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k$

set of

by solving $k+1$ simultaneous equations

which are called normal equations

(see page 636)

Then fitted model is

\[ \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \ldots + \hat{\beta}_k x_k \]

See also an example 12.5 p. 637