Recurrence relation for central moments for the Binomial \((n, p)\) distn:

\[
\mu_{x+1} = p q \left[ \frac{\mu_x}{n} \mu_{x-1} + \frac{d\mu_x}{dp} \right]
\]

\[
\mu_x = E \left[ (x - np)^x \right] = \sum_{x=0}^{n} (x - np)^x \binom{n}{x} p^x q^{n-x}
\]

\[
\frac{d\mu_x}{dp} = \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} (x - np)^x \left\{ xq - p(n-x) \right\}
\]

\[
+ \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} (x - np)^x \left\{ (x-1)q - x(n-x)p \right\}
\]

\[
+ \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} (x - np)^x \left\{ -nq \right\}
\]

\[
N \cdot \frac{d\mu_x}{dp} = -1
\]

\[
= \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ xq - p(n-x) \right\}
\]

\[
+ \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ (x-1)q - x(n-x)p \right\}
\]

\[
+ \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ -nq \right\}
\]

\[
= \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ xq - p(n-x) \right\}
\]

\[
- \frac{d\mu_x}{dp} = \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ xq - p(n-x) \right\}
\]

\[
p q \frac{d\mu_x}{dp} = \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} (x - np)^x \left\{ xq - p(n-x) \right\}
\]

\[
= \sum_{x=0}^{n} \binom{n}{x} x^{x-1} p^x q^{n-x} \left\{ \frac{x}{n} \left[ \mu_x \right]_{x-1} \right\}
\]

\[
\Rightarrow \mu_{x+1} = p q \left[ \frac{\mu_x}{n} \mu_{x-1} + \frac{d\mu_x}{dp} \right]
\]
3.1a. Let $X$ be a random variable with a binomial distribution. Find $P(X = n)$ where $n = 2$.

3.1b. Let $X$ be a random variable with a binomial distribution. Find $P(X = n)$ where $n = 3$.

3.1c. Let $X$ be a random variable with a binomial distribution. Find $P(X = n)$ where $n = 4$.

3.2. The mean of a random variable $X$ is $\mu$. Show that $E(X^2) = \mu^2 + \sigma^2$.

3.3. The variance of a random variable $X$ is $\sigma^2$. Show that $E(X^2) = \mu^2 + \sigma^2$.

The probability of each

From the four mutually exclusive ways that $X$ and $X'$ did not come up.

3.7. Let $X$ be a random variable with a binomial distribution with parameters $n$ and $p$. For $0 < x < 1$, the probability of exactly $x$ successes in $n$ independent trials is given by the binomial distribution.

3.8. Let $X$ be a random variable with a binomial distribution with parameters $n$ and $p$. Find the probability of exactly $x$ successes in $n$ independent trials.

3.9. Let $X$ be a random variable with a binomial distribution. Find the probability of exactly $x$ successes in $n$ independent trials.

3.10. Let $X$ be a random variable with a binomial distribution. Find the probability of exactly $x$ successes in $n$ independent trials.

EXERCISES

For all real values of $\mu$, $\mu^2$, $\mu^3$, $\mu^4$, $\mu^5$, $\mu^6$, $\mu^7$, and $\mu^8$, show that for each one-variable marginal and each two-variable marginal $P(X = n)$ is binomial, each two-variable marginal $P(Y = m)$ is binomial, and $P(X = n, Y = m)$ is binomial.