Example: Conf. Int. & Test of hypotheses for \((\mu_1 - \mu_2)\)

Sample 1: Sample 2

\(n_1 = 53\) \(n_2 = 51\) (large samples)

\(\bar{x}_1 = 0.35\) \(\bar{x}_2 = 0.340\)

\(s_1 = 1.046\) \(s_2 = 0.960\)

Find 90% Conf. Int. for \((\mu_1 - \mu_2)\)

a) 90% Confidence Interval for \((\mu_1 - \mu_2)\)

\[ (\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \]

\[ 1 - \alpha = 0.90 \]

\[ \alpha = 0.10 \]

we will use estimates \(\hat{s}_1^2\) for \(\sigma_1^2\) and \(\hat{s}_2^2\) for \(\sigma_2^2\)

\[ Z_{0.05} = Z_{0.05} = 1.645 \]

\[ s_1^2 = (1.046)^2 = 1.0941 \]

\[ s_2^2 = (0.960)^2 = 0.9216 \]

\[ (0.35 - 0.340) \pm 1.645 \sqrt{1.0941/53 + 0.9216/51} \]

\[ -0.05 \pm 1.645 \sqrt{0.0206 + 0.0181} \]

\[ = -0.305 \pm 1.645 \sqrt{0.0387} = -0.305 \pm 1.645(0.197) \]

\[ = -0.305 \pm 0.3317 \]

\[ = -0.6367 \leq \mu_1 - \mu_2 \leq 0.0267 \]

We are 90% confident that \((\mu_1 - \mu_2)\) is between -0.6367 and 0.0267
b) Test of hypothesis

(1) $H_0: \mu_1 = \mu_2 = 0$

$H_a: \mu_1 \neq \mu_2$ (Two sided test)

(2) $\alpha = 0.10$

(3) Test Stat: $Z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

(4) Rule: Reject $H_0$ if $|Z| > Z_{0.05}$ $\alpha = 0.10$

$\bar{x}_1 = 0.197$

(5) Use sample data to calculate

$Z = \frac{(0.035 - 0.305) - 0}{\sqrt{0.0941 + 0.9216}}$

$\Rightarrow Z = \frac{-0.27}{.41} = -0.6548$ (See previous calculation)

(6) Conclusion: Calculated $Z = -0.6548$

$|Z| = 0.6548 < 1.645$ which is not greater than 1.645 $\Rightarrow$ Do not reject $H_0$

$\Rightarrow$ not sufficient to conclude that $\mu_1$ is different from $\mu_2$. 