MAP 2302 WRITTEN HOMEWORK #1

Question 1. Find a solution to the differential equation.

$$xy' = 17y$$

(Hint: Try a 'power function' of the form $y = x^r$ where r is a constant.)

Following the hint, try $y = x^r$

$\downarrow$

$y' = r \cdot x^{r-1}$

Sub into given equation:

$$x \cdot \underbrace{r x^{r-1}}_{y'} = 17 \underbrace{x^r}_{y}$$

$$r \cdot x^r = 17 x^r$$

So $r = 17$ works

$y = x^{17}$ is a solution.

Question 2. Consider the following differential equation.

$$y'''' = y$$

Note: In case it's hard to read, y'''' is the fourth derivative. This is also sometimes written as $y^{(4)}$ or $\frac{d^4y}{dx^4}$. So in this case, we're looking for functions whose fourth derivative is equal to the original function.

Which of the following are solutions to the differential equation?

(i) $y = \sin x$ YES
(ii) $y = \cos x$ YES
(iii) $y = e^x$ YES
(iv) $y = \ln x$ NO
(v) $y = x^4$ NO
(vi) $y = 17\sin x + 83\cos x - 221e^x$ YES

(i) $y = \sin x$
$y' = \cos x$
$y'' = -\sin x$
$y''' = -\cos x$
$y'''' = \sin x = y$, so this is a solution.

(ii) $y = \cos x$
$y' = -\sin x$
$y'' = -\cos x$
$y''' = \sin x$
$y'''' = \cos x = y$, so this is a solution.

(iii) $y = e^x$

$y' = e^x$

$y'' = e^x$

$y''' = e^x$

$y'''' = e^x = y$, so this is a solution.

(iv) $y = \ln x$

$y' = \frac{1}{x} = x^{-1}$

$y'' = -1x^{-2}$

$y''' = +2x^{-3}$

$y'''' = -6x^{-4}$ which is not equal to y. Not a solution.

(v) $y = x^4$

$y' = 4x^3$

$y'' = 12x^2$

$y''' = 24x$

$y'''' = 24$ which is not equal to y. Not a solution.

(vi) $y = 17\sin x + 83\cos x - 221e^x$

$y' = 17\cos x - 83\sin x - 221e^x$

$y'' = -17\sin x - 83\cos x - 221e^x$

$y''' = -17\cos x + 83\sin x - 221e^x$

$y'''' = 17\sin x + 83\cos x - 221e^x = y$

This is a solution.