MAP 2302 WRITTEN HOMEWORK #2

Question 1. Find the general solution of the differential equation.

$$\frac{dy}{dx} = \frac{y}{x} + 17x + 83$$

Can be rewritten as

$$\circledast \quad y' + \underbrace{\left(-\frac{1}{x}\right)}_{P(x)} y = \underbrace{17x + 83}_{Q(x)} \quad \text{which is LINEAR.}$$

Find integrating factor. $\int P(x)\,dx = \int -\frac{1}{x}\,dx = -\ln|x| + C$

Can use $-\ln x$

$$e^{\int P(x)dx} = e^{-\ln x} = 1/x$$

So multiply both sides of $\circledast$ by $\frac{1}{x}$.

$$\frac{1}{x}y' + \frac{-1}{x^2}y = 17 + \frac{83}{x}$$

$$\frac{d}{dx}\left(\frac{1}{x}y\right) = 17 + \frac{83}{x}$$

$$\frac{1}{x}y = 17x + 83\ln|x| + C$$

$$y = 17x^2 + 83x\ln|x| + Cx$$

Question 2. Consider the following differential equation.

$$\frac{dy}{dx} + x^5 y = 0$$

(i) Is it separable?

(ii) Is it linear?

(iii) Find the general solution using any correct method.

(i) Yes. We can rearrange as $\frac{dy}{dx} = -x^5 y \Rightarrow \frac{dy}{y} = -x^5 dx$

(ii) Yes. It has the form $\frac{dy}{dx} + P(x) \cdot y = Q(x)$ where $P(x) = x^5$, $Q(x) = 0$

(iii) METHOD 1: SEPARATION OF VARIABLES

$$\frac{dy}{y} = -x^5 dx \Rightarrow \int \frac{dy}{y} = \int -x^5 dx \Rightarrow \ln|y| = -\frac{x^6}{6} + C$$

$$\Rightarrow |y| = e^{-x^6/6 + C} = e^{-x^6/6} \cdot \underbrace{e^C}_{\text{rename}} = C \cdot e^{-x^6/6}$$

$$\Rightarrow y = \underbrace{\pm C}_{\text{rename}} \cdot e^{-x^6/6} \Rightarrow \boxed{y = C \cdot e^{-x^6/6}}$$

METHOD 2: FIND INTEGRATING FACTOR FOR LINEAR 1ST ORDER D.E.

$$P(x) = x^5 \Rightarrow \int P(x)\,dx = \frac{x^6}{6} \Rightarrow e^{\int P(x)dx} = e^{x^6/6}$$

$$e^{x^6/6} \cdot (y' + x^5 y) = e^{x^6/6} \cdot 0 \Rightarrow e^{x^6/6} y' + \underbrace{e^{x^6/6} x^5}_{\text{deriv. of } e^{x^6/6}} y = 0$$

$$\frac{d}{dx}\left(e^{x^6/6} y\right) = 0 \Rightarrow e^{x^6/6} y = C \Rightarrow \boxed{y = Ce^{-x^6/6}}$$