# MAP 2302 WRITTEN HOMEWORK #4

**Question 1.** Given the differential equation

$$y'' + 2y' + 5y = -50\sin 5t$$

find a solution of the form $y = A\cos 5t + B\sin 5t$.

$$y' = -5A\sin 5t + 5B\cos 5t$$

$$y'' = -25A\cos 5t - 25B\sin 5t$$

Left side $= y'' + 2y' + 5y$

$= -25A\cos 5t - 25B\sin 5t$
$\quad + 2(5B\cos 5t - 5A\sin 5t)$
$\quad + 5(A\cos 5t + B\sin 5t)$

$= (-25A + 10B + 5A)\cos 5t + (-25B - 10A + 5B)\sin 5t$

$$= (\underbrace{-20A + 10B}_{\text{Need this to be } 0})\cos 5t + (\underbrace{-10A - 20B}_{\text{Need this to be } -50})\sin 5t$$

(i) $-20A + 10B = 0 \xrightarrow{\times 2} -40A + 20B = 0$

(ii) $-10A - 20B = -50$

$$\begin{array}{r} -40A + 20B = 0 \\ -10A - 20B = -50 \\ \hline -50A \quad = -50 \end{array}$$

$A = 1$

$\Rightarrow B = 2$

$$y = \cos 5t + 2\sin 5t$$

**Question 2.** Solve the initial value problem.

$$y'' + 2y' + y = 0, \qquad y(0) = 1, \qquad y'(0) = -3$$

Auxiliary equation $r^2 + 2r + 1 = 0$

$$(r+1)^2 = 0$$

$r = -1$ is a repeated root

Two independent solutions will be $e^{-t}$ and $te^{-t}$

$y = Ae^{-t} + Bte^{-t}$ Use initial conditions to find A and B

$\downarrow$

$y' = -Ae^{-t} + Be^{-t} - Bte^{-t}$

$y(0) = A$ Must have $A = 1$

$y'(0) = -A + B$ $-A + B = -3$

$\Rightarrow B = -2$

Solution: $\boxed{y = e^{-t} - 2te^{-t}}$