

MAP 2302 WRITTEN HOMEWORK #5

Question 1. Solve the initial value problem.

$$y'' - 8y' + 17y = 0, \quad y(0) = 3, \quad y'(0) = 17$$

Auxiliary eqn: $r^2 - 8r + 17 = 0$

$$r^2 - 8r + 16 = -1$$

$$(r-4)^2 = -1$$

$$r - 4 = \pm i$$

$$r = 4 \pm i$$

$\alpha \pm \beta i$
 $\alpha = 4, \beta = 1$

\Rightarrow General solution is $y = Ae^{4t} \cos t + Be^{4t} \sin t$

$\Rightarrow y' = 4Ae^{4t} \cos t - Ae^{4t} \sin t + 4Be^{4t} \sin t + Be^{4t} \cos t$

$y(0) = \underbrace{Ae^0}_{1} \underbrace{\cos 0}_{1} + \underbrace{Be^0}_{0} \underbrace{\sin 0}_{0} = A \Rightarrow A = 3$

$y'(0) = 4A \underbrace{e^0}_{1} \underbrace{\cos 0}_{1} - A \underbrace{e^0}_{0} \underbrace{\sin 0}_{0} + 4B \underbrace{e^0}_{0} \underbrace{\sin 0}_{0} + B \underbrace{e^0}_{1} \underbrace{\cos 0}_{1}$

$= 4A + B \Rightarrow \underbrace{4A + B}_{12} = 17 \Rightarrow B = 5$

Answer: $y = 3e^{4t} \cos t + 5e^{4t} \sin t$

or $e^{4t} (3 \cos t + 5 \sin t)$

Question 2. Find the general solution to the differential equation.

$$y'' - 4y' - 12y = 3e^{5t}$$

Auxiliary eqn for homog DE: $r^2 - 4r - 12 = 0$

$$(r-6)(r+2) = 0 \quad r=6, r=-2$$

General solution of homog DE: $y = C_1 e^{6t} + C_2 e^{-2t}$

Next, find a particular solution of the nonhomogeneous DE.

Try a solution of the form $y = Ae^{5t}$

$$\downarrow$$
$$y' = 5Ae^{5t}$$

$$\downarrow$$
$$y'' = 25Ae^{5t}$$

Plug into DE: $\underbrace{25Ae^{5t}}_{y''} - 4\underbrace{(5Ae^{5t})}_{y'} - 12\underbrace{(Ae^{5t})}_y = 3e^{5t}$

$$(25A - 20A - 12A)e^{5t} = 3e^{5t}$$

$$-7Ae^{5t} = 3e^{5t} \Rightarrow A = -\frac{3}{7}$$

A particular solution of the nonhomog DE is $y = -\frac{3}{7}e^{5t}$.

The general solution to the nonhomog DE is

$$y = C_1 e^{6t} + C_2 e^{-2t} - \frac{3}{7} e^{5t}$$