

MAP 2302 WRITTEN HOMEWORK #6

Question 1. Find the general solution of the differential equation.

$$y'' - 2y' - 3y = 3t^2 + t + 5 \quad \text{Nonhomogeneous}$$

Associated homog. DE is $y'' - 2y' - 3y = 0$. Aux. eqn. $r^2 - 2r - 3 = 0$
 $(r+1)(r-3) = 0$

General soln of homog DE is $y = c_1 e^{-t} + c_2 e^{3t}$. $r = -1, r = 3$

Now find a particular soln of the nonhomog DE.

Try $y = At^2 + Bt + C$ } \rightarrow Plug these into the DE:

$$\left. \begin{array}{l} y' = 2At + B \\ y'' = 2A \end{array} \right\} \begin{array}{l} 2A - 2(2At + B) - 3(At^2 + Bt + C) = 3t^2 + t + 5 \\ \underbrace{\hspace{1cm}}_{y''} \quad \underbrace{\hspace{1cm}}_{y'} \quad \underbrace{\hspace{1cm}}_y \end{array}$$

$$2A - 4At - 2B - 3At^2 - 3Bt - 3C = 3t^2 + t + 5$$

$$\Rightarrow (-3A)t^2 + (-4A - 3B)t + (2A - 2B - 3C) = 3t^2 + t + 5$$

$$\Rightarrow \text{(i) } -3A = 3 \implies A = -1$$

$$\text{(ii) } -4A - 3B = 1 \implies 4 - 3B = 1 \implies B = 1$$

$$\text{(iii) } 2A - 2B - 3C = 5 \implies -2 - 2 - 3C = 5 \implies -3C = 9 \implies C = -3$$

Particular soln is $y = -t^2 + t - 3$

Final answer: $y = c_1 e^{-t} + c_2 e^{3t} - t^2 + t - 3$

Question 2. Solve the initial value problem.

$$y' - y = 82, \quad y(0) = 1$$

Nonhomogeneous

can also solve as
 $y' = y$ or $\frac{dy}{dt} = y$

Associated homogeneous DE is $y' - y = 0 \Rightarrow$ aux eqn $r - 1 = 0$
General solution of homog DE is $y = Ce^t$.

To find a particular solution of the nonhomogeneous DE,

try $y = A$
 $y' = 0$ } Plug into the DE: $0 - A = 82 \Rightarrow A = -82$

Hence the general solution of the nonhomog DE is: $y = Ce^t - 82$

Next, use the initial condition to find C .

$$y(t) = Ce^t - 82$$

$$y(0) = \underbrace{Ce^0}_{1} - 82 = C - 82. \quad \text{But we must have } y(0) = 1.$$

$$\text{So } C - 82 = 1$$

$$\Rightarrow C = 83$$

So the solution to the initial value problem is

$$y = 83e^t - 82$$