MAP 2302 WRITTEN HOMEWORK #10

Question 1. Consider the differential equation $x^2y'' + 5xy' + 4y = 0$. By substituting a proposed solution of the form $y = x^r$ (and its derivatives), show that r must be -2.

$$y = x^r \Rightarrow y' = rx^{r-1} \Rightarrow y'' = r(r-1)x^{r-2}$$

Plug into DE: $x^2 \cdot r(r-1)x^{r-2} + 5x \cdot rx^{r-1} + 4 \cdot x^r = 0$

$$x^r\left(r(r-1) + 5r + 4 \right) = 0$$

$$r^2 - r + 5r + 4 = 0$$

$$r^2 + 4r + 4 = 0$$

$$(r+2)^2 = 0$$

$$r = -2$$

$4y + xy$

Question 2. Consider the differential equation $x^2y'' + 5xy' + \underbrace{(4+x)y}_{} = 0$. If x is near 0, this resembles the equation in Question 1. Find the first five terms of a solution that looks like x^{-2} times a power series.

$$y = x^{-2}\cdot(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots)$$

$$y = a_0x^{-2} + a_1x^{-1} + a_2 + a_3x + a_4x^2 + \dots$$

$$y' = -2a_0x^{-3} - a_1x^{-2} + a_3 + 2a_4x + \dots$$

$$y'' = 6a_0x^{-4} + 2a_1x^{-3} + 2a_4 + \dots$$

$$x^2y'' = 6a_0x^{-2} + 2a_1x^{-1} + 0 + 0 + 2a_4x^2 + \dots$$

$$5xy' = -10a_0x^{-2} - 5a_1x^{-1} + 0 + 5a_3x + 10a_4x^2 + \dots$$

$$4y = 4a_0x^{-2} + 4a_1x^{-1} + 4a_2 + 4a_3x + 4a_4x^2 + \dots$$

$$xy = a_0x^{-1} + a_1 + a_2x + a_3x^2 + \dots$$

ADD

$$0 = 0x^{-2} + (a_1 + a_0)x^{-1} + (4a_2 + a_1) + (9a_3 + a_2)x + (16a_4 + a_3)x^2 + \dots$$

$$\Rightarrow a_1 + a_0 = 0 \Rightarrow a_1 = -a_0$$

$$4a_2 + a_1 = 0 \Rightarrow a_2 = -\frac{1}{4}a_1 = \frac{1}{4}a_0$$

$$9a_3 + a_2 = 0 \Rightarrow a_3 = -\frac{1}{9}a_2 = -\frac{1}{36}a_0$$

$$16a_4 + a_3 = 0 \Rightarrow a_4 = -\frac{1}{16}a_3 = \frac{1}{576}a_0$$

$$y = a_0x^{-2} - a_0x^{-1} + \frac{1}{4}a_0 - \frac{1}{36}a_0x + \frac{1}{576}a_0x^2 + \dots$$