MAP 2302 WRITTEN HOMEWORK #11

Question 1. Let f be the piecewise function defined as follows.

$$f(t) = \begin{cases} 0 & \text{if } t < 17 \\ 1 & \text{if } t \geq 17 \end{cases}$$

Find the Laplace transform of $f(t)$ using the definition.

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^{17} e^{-st} f(t)\,dt + \int_{17}^\infty e^{-st} f(t)\,dt$$

$$= \underbrace{\int_0^{17} e^{-st} \cdot 0\,dt}_{=0} + \int_{17}^\infty e^{-st} \cdot 1\,dt$$

$$= \int_{17}^\infty e^{-st}\,dt = \left[\frac{e^{-st}}{-s}\right]_{t=17}^{t=\infty} = \left[\frac{e^{-st}}{s}\right]_{t=\infty}^{t=17}$$

$$= \frac{e^{-17s}}{s} - \frac{e^{-s\cdot\infty}}{s} = \frac{e^{-17s}}{s} - \frac{0}{s} \quad \text{if } s>0$$

$$= \frac{e^{-17s}}{s} \quad (\text{if } s \text{ is positive})$$

Question 2. Find the Laplace transform of the function $f(t) = \sin 2t \sin 5t$. (Hint: You can use trigonometric identities, as well as Laplace transforms of known functions.)

We can use the following trig identity ("product-to-sum" identity)

$$\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

Therefore

$$\sin 5t \sin 2t = \frac{1}{2}\cos 3t - \frac{1}{2}\cos 7t$$

Therefore

$$\mathcal{L}\{\sin 5t \sin 2t\} = \mathcal{L}\left\{\frac{1}{2}\cos 3t - \frac{1}{2}\cos 7t\right\}$$

$$= \frac{1}{2}\mathcal{L}\{\cos 3t\} - \frac{1}{2}\mathcal{L}\{\cos 7t\}$$

$$= \frac{1}{2}\cdot\frac{s}{s^2+9} - \frac{1}{2}\cdot\frac{s}{s^2+49}$$