WRITE YOUR NAME:

MAP 2302 Quiz 16
Thursday October 31st

Find the first four nonzero terms in the Taylor series for the solution to the initial value problem.

$$y' = x^2 + y^2, \qquad y(0) = 1$$

$$y'' = 2x + 2y \cdot y' \quad \text{(CHAIN RULE!)}$$

$$y''' = 2 + 2y' \cdot y' + 2y \cdot y'' \quad \text{(PRODUCT RULE!)}$$

$$y(0) = 1$$

$$y'(0) = 0^2 + y(0)^2 = 0 + 1^2 = 0 + 1 = 1$$

$$y''(0) = 2 \cdot 0 + 2y(0)\, y'(0) = 0 + 2 \cdot 1 \cdot 1 = 2$$

$$y'''(0) = 2 + 2y'(0)\, y'(0) + 2y(0)\, y''(0)$$

$$= 2 + 2 \cdot 1 \cdot 1 + 2 \cdot 1 \cdot 2 = 2 + 2 + 4 = 8$$

First four nonzero terms are $y(0) + y'(0)x + \dfrac{y''(0)}{2!} x^2 + \dfrac{y'''(0)}{3!} x^3$

$$= 1 + 1x + \frac{2}{2!} x^2 + \frac{8}{3!} x^3 \quad \text{or} \quad 1 + x + \frac{2}{2} x^2 + \frac{8}{6} x^3$$

$$\text{or} \quad 1 + x + x^2 + \frac{4}{3} x^3$$