

WRITE YOUR NAME:

MAP 2302 Quiz 18  
Thursday November 7th

three (sorry)

**EITHER** write the solution to the initial value problem as a power series (find the first ~~four~~ nonzero terms) **OR** solve the initial problem exactly, using techniques from earlier in the course.

$$y' = 2xy, \quad y(0) = 1$$

Power series?  $y'' = (2x)'y + 2x(y)'$  ← Product again

$$y''' = 2y' + 2y' + 2xy'' = 4y' + 2xy''$$

$$y^{(4)} = 4y'' + 2y'' + 2xy''' = 6y'' + 2xy'''$$

$$y(0) = 1, \quad y'(0) = 2 \cdot 0 \cdot y(0) = 0, \quad y''(0) = 2y(0) + 2 \cdot 0 \cdot y'(0) = 2 \cdot 1 = 2,$$

$$y'''(0) = 4y'(0) + 2 \cdot 0 \cdot y''(0) = 4 \cdot 0 = 0, \quad y^{(4)}(0) = 6y''(0) + 2 \cdot 0 \cdot y'''(0) = 6 \cdot 2 = 12$$

$$y = y(0) + \underbrace{y'(0)}_0 x + \frac{y''(0)}{2!} x^2 + \frac{y'''(0)}{3!} x^3 + \frac{y^{(4)}(0)}{4!} x^4$$

$$= 1 + \frac{2}{2!} x^2 + \frac{12}{4!} x^4 \text{ or } 1 + x^2 + \frac{x^4}{2}$$

Exact soln: Separable.  $\frac{dy}{dx} = 2xy \Rightarrow \frac{1}{y} dy = 2x dx \Rightarrow \int \frac{1}{y} dy = \int 2x dx$

$$\Rightarrow \ln|y| = x^2 + C \Rightarrow |y| = e^{x^2+C} = e^C \cdot e^{x^2} \Rightarrow y = \underbrace{\pm e^C}_{\text{can rename}} \cdot e^{x^2}$$

Gen soln is  $y = Ke^{x^2}$ . Can use initial condition to find K.

$$\text{If } x=0 \text{ then } y=1. \quad 1 = Ke^{0^2} = K \cdot 1 \Rightarrow K=1$$

$$y = e^{x^2}. \quad \text{Note: As a series, this is } 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \dots$$