

WRITE YOUR NAME:

MAP 2302 Quiz 19  
Tuesday November 12th

Solve the initial value problem using any correct method.

$$y'' + y = 2 + t^2, \quad y(0) = 0, \quad y'(0) = 0$$

METHOD 1. (i) Corresponding homog DE is  $y'' + y = 0$   
We've previously seen general soln of this homog DE  
is  $y = c_1 \sin t + c_2 \cos t$

(ii) Try a particular soln of the form  $y = At^2 + Bt + C$

$$\Rightarrow y' = 2At + B \Rightarrow y'' = 2A. \text{ Plug into DE: } \underbrace{2A}_{y''} + \underbrace{At^2 + Bt + C}_y = 2 + t^2$$

$$\Rightarrow \underbrace{At^2}_{\text{w}} + \underbrace{Bt}_{\text{ce}} + \underbrace{(2A+C)}_{\text{////}} = \underbrace{1t^2}_{\text{w}} + \underbrace{0t}_{\text{ce}} + \underbrace{2}_{\text{////}}$$

$$\Rightarrow \left. \begin{array}{l} A = 1 \\ B = 0 \\ 2A + C = 2 \Rightarrow 2 + C = 2 \Rightarrow C = 0 \end{array} \right\} \Rightarrow y = 1t^2 + 0t + 0 = t^2$$

is a particular soln

(i) & (ii)  $\Rightarrow$  general soln of nonhomog DE is  $y = c_1 \sin t + c_2 \cos t + t^2$

(iii) Use initial conditions to find  $c_1$  and  $c_2$

$$y(t) = c_1 \sin t + c_2 \cos t + t^2 \Rightarrow y(0) = c_1 \underbrace{\sin 0}_0 + c_2 \underbrace{\cos 0}_1 + 0^2 = c_2$$

$$y'(t) = c_1 \cos t - c_2 \sin t + 2t \Rightarrow y'(0) = c_1 \underbrace{\cos 0}_1 - c_2 \underbrace{\sin 0}_0 + 2 \cdot 0 = c_1$$

$$y(0) = 0 \Rightarrow c_2 = 0$$

$$y'(0) = 0 \Rightarrow c_1 = 0$$

FINAL ANSWER:  $y = t^2$   
( $y = 0 \sin t + 0 \cos t + t^2$ )