Recall **Taylor series** from Calc II.

Examples of well-known Taylor series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Fact: These three series converge for all values of $x$.

Some other series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$
$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Fact: These three series converge for only some values of $x$.

Fact: The Taylor series centered at 0 for any function $y$ looks like

$$y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \cdots$$

The Taylor series centered at $a$ looks like

$$y(a) + y'(a)(x-a) + \frac{y''(a)}{2!}(x-a)^2 + \frac{y'''(a)}{3!}(x-a)^3 + \cdots$$

Some differential equations are difficult to solve explicitly, but we might be able to write the solution as an infinite series.

**WEEK 8 EXAMPLE 1:** Solve the initial value problem.

$$y' = y + \frac{1}{x}, \qquad y(1) = 1$$

(This is linear. What happens if we try to solve it using an integrating factor?)

**WEEK 8 EXAMPLE 2:** Solve the initial value problem.

$$y' = x^2 + y^2, \qquad y(0) = 1$$

(This is nonlinear. Can it be solved using techniques from this course?)

**WEEK 8 EXAMPLE 3:** Solve the initial value problem.

$$y'' = xy' + y, \qquad y(0) = 1, \qquad y'(0) = 0$$

What if we don't have an initial value problem?

**WEEK 8 EXAMPLE 4:** Find the general solution of the differential equation.

$$y'' = xy' + y$$