Method of Frobenius

Some differential equations 'resemble' a Cauchy-Euler equation.

Cauchy-Euler: $ax^2y'' + bxy' + cy = 0$ where a, b, c are *constants*

This will have solutions of the form $y = x^r$ where r is a root of the 'characteristic equation' $ar(r-1) + br + c = 0$.

Now consider the following differential equation.

$$9x^2y'' + 9x^2y + 2y = 0 \tag{1}$$

This is *not* Cauchy-Euler, but it can be written as

$$a(x) \cdot x^2y'' + b(x) \cdot xy' + c(x) \cdot y = 0$$

where $a(x), b(x), c(x)$ are *functions*. Specifically $a(x) = 9$, $b(x) = 9x$, and $c(x) = 2$.

When $x \approx 0$, we have $a(x) \approx 9$, $b(x) \approx 0$, and $c(x) \approx 2$.

Therefore for $x \approx 0$, the differential equation (1) 'resembles' the Cauchy-Euler equation

$$9x^2y'' + 0y' + 2y = 0$$

whose characteristic equation is $9r(r-1) + 0r + 2 = 0$.

We can find a solution to (1) of the form x^r times a power series!