Review of trigonometry

The website BetterExplained.com has an informal review of the concepts of trigonometry.

Special values of sine, cosine, and tangent in the first quadrant are summarized below.

You also need to know how the trig functions are extended to the other three quadrants.

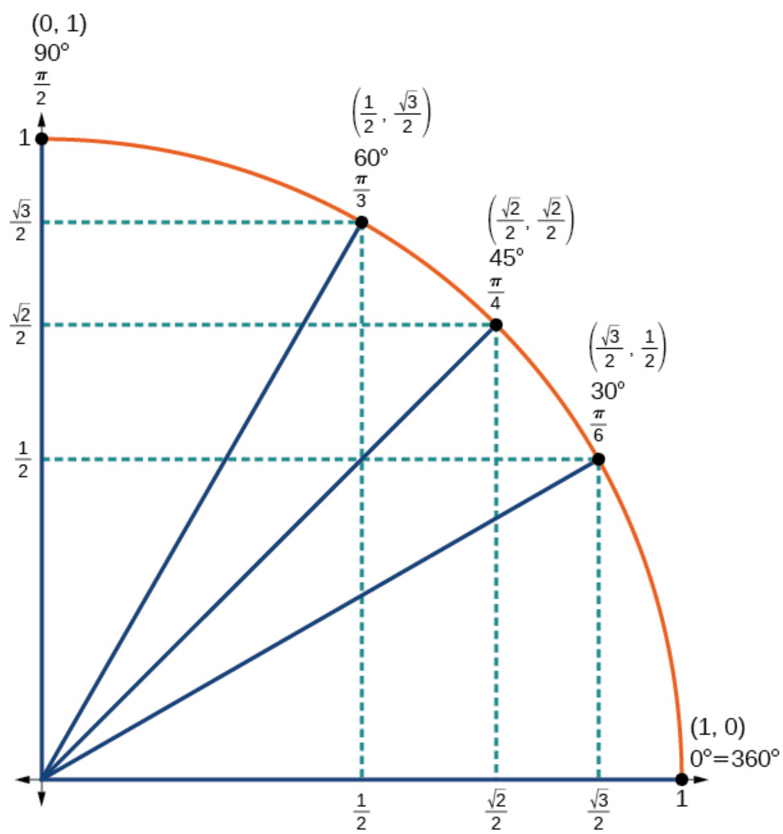


Image from Chapter 5 of Precalculus 1e (OpenStax)

θ	$\sin\theta$	$\cos\theta$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	undefined

Also remember: $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$

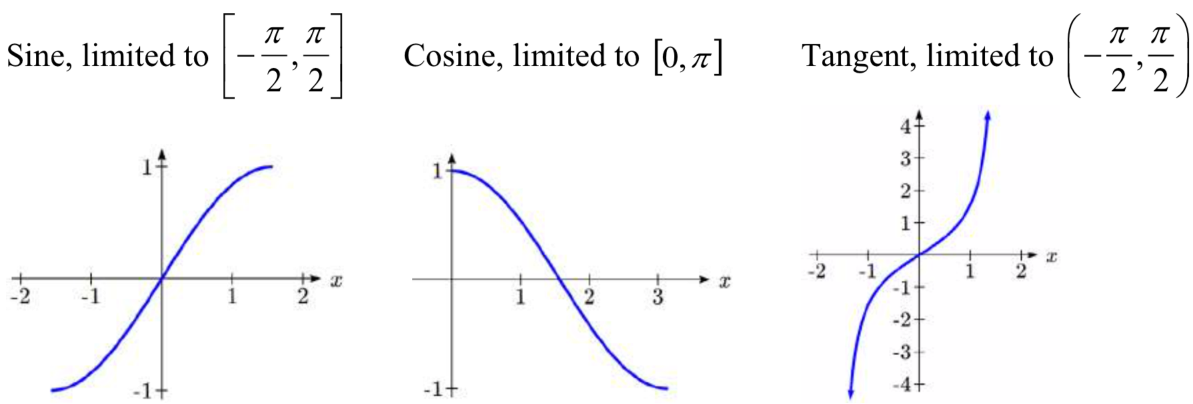
Restricting the trig functions

To define *inverse* trig functions, we first restrict the domains of the *regular* trig functions.

Restrict sine so its **inputs** are in $[-\pi/2, \pi/2]$ and each **output** in $[-1, 1]$ occurs once.

Restrict cosine so its **inputs** are in $[0, \pi]$ and each **output** in $[-1, 1]$ occurs once.

Restrict tangent so its **inputs** are in $(-\pi/2, \pi/2)$ and each **output** in $(-\infty, \infty)$ occurs once.



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θ	$\sin\theta$
$-\pi/2$	-1
$-\pi/3$	$-\sqrt{3}/2$
$-\pi/4$	$-\sqrt{2}/2$
$-\pi/6$	$-1/2$
0	0
$\pi/6$	$1/2$
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1

θ	$\cos\theta$
0	1
$\pi/6$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$1/2$
$\pi/2$	0
$2\pi/3$	$-1/2$
$3\pi/4$	$-\sqrt{2}/2$
$5\pi/6$	$-\sqrt{3}/2$
π	-1

θ	$\tan\theta$
$-\pi/2$	$-\infty$
$-\pi/3$	$-\sqrt{3}$
$-\pi/4$	-1
$-\pi/6$	$-1/\sqrt{3}$
0	0
$\pi/6$	$1/\sqrt{3}$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$
$\pi/2$	∞

(Technically, the ∞ and $-\infty$ in the table for $\tan\theta$ should be rephrased using limits.)

After restricting the trig functions as above, we can then define the **inverse** trig functions.

Inverse trigonometric functions

If a function is one-to-one, we get the inverse function by exchanging inputs and outputs.

The **inverse** sine (or arcsin) has **inputs** in $[-1, 1]$ and **outputs** in $[-\pi/2, \pi/2]$.

The **inverse** cosine (or arccos) has **inputs** in $[-1, 1]$ and **outputs** in $[0, \pi]$.

The **inverse** tangent (or arctan) has **inputs** in $(-\infty, \infty)$ and **outputs** in $(-\pi/2, \pi/2)$.

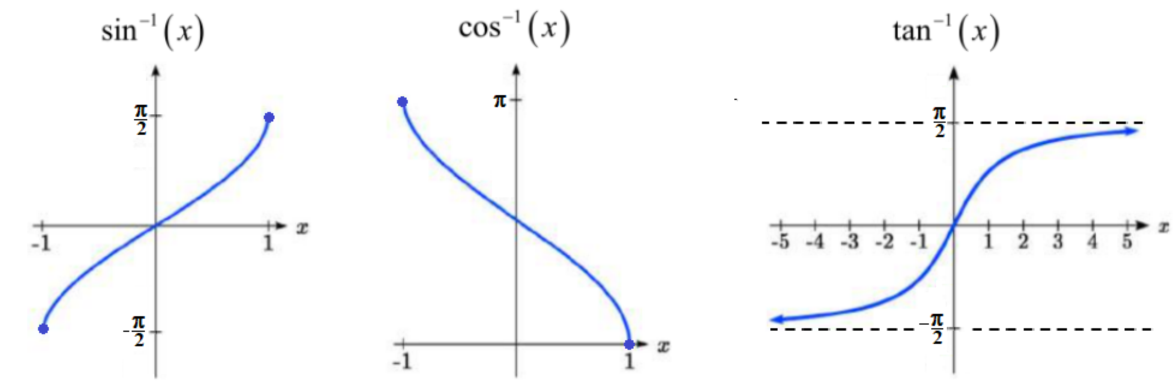

Image from Monroe Community College College Algebra / Precalculus

x	$\arcsin x$
-1	$-\pi/2$
$-\sqrt{3}/2$	$-\pi/3$
$-\sqrt{2}/2$	$-\pi/4$
$-1/2$	$-\pi/6$
0	0
$1/2$	$\pi/6$
$\sqrt{2}/2$	$\pi/4$
$\sqrt{3}/2$	$\pi/3$
1	$\pi/2$

x	$\arccos x$
-1	π
$-\sqrt{3}/2$	$5\pi/6$
$-\sqrt{2}/2$	$3\pi/4$
$-1/2$	$2\pi/3$
0	$\pi/2$
$1/2$	$\pi/3$
$\sqrt{2}/2$	$\pi/4$
$\sqrt{3}/2$	$\pi/6$
1	0

x	$\arctan x$
$-\infty$	$-\pi/2$
$-\sqrt{3}$	$-\pi/3$
-1	$-\pi/4$
$-1/\sqrt{3}$	$-\pi/6$
0	0
$1/\sqrt{3}$	$\pi/6$
1	$\pi/4$
$\sqrt{3}$	$\pi/3$
∞	$\pi/2$

(Again, technically the ∞ and $-\infty$ in the third table should be rephrased using limits.)

Review of logarithmic functions

Logarithmic functions are the inverse functions of exponential functions.

To understand logarithms intuitively, it may help to look at a 'round' base such as 10.

x	10^x
-6	0.000 001
-5	0.000 01
-4	0.000 1
-3	0.001
-2	0.01
-1	0.1
0	1
1	10
2	100
3	1000
4	10 000
5	100 000
6	1 000 000

x	$\log x$
0.000 001	-6
0.000 01	-5
0.000 1	-4
0.001	-3
0.01	-2
0.1	-1
1	0
10	1
100	2
1000	3
10 000	4
100 000	5
1 000 000	6

That can help you develop a *feeling* for the rules of logarithms, and *why* they are true.

RULE	**EXAMPLE**
$\log(ab) = \log(a) + \log(b)$	$\underbrace{\log(100 \cdot 1000)}_{\log(100\,000)=5} = \underbrace{\log(100)}_{2} + \underbrace{\log(1000)}_{3}$
$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$	$\underbrace{\log\left(\frac{100\,000}{100}\right)}_{\log(1000)=3} = \underbrace{\log(100\,000)}_{5} - \underbrace{\log(100)}_{2}$
$\log(a^r) = r \cdot \log(a)$	$\underbrace{\log(100^3)}_{\log(1\,000\,000)=6} = 3 \cdot \underbrace{\log(100)}_{2}$

You can also remember those rules of logarithms using words.

- Logs change multiplying into adding. (Log of product equals sum of logs.)
- Logs change dividing into subtracting. (Log of quotient equals difference of logs.)
- Logs change exponents into constant multiples.

The natural logarithm function

The natural logarithm $\ln x$ is the base e logarithm, where $e \approx 2.718$ is a special constant.

x	e^x
-1	$1/e \approx 0.37$
0	1
1	$e \approx 2.7$
2	$e^2 \approx 7.4$
3	$e^3 \approx 20$

x	$\ln x$
$1/e$	-1
1	0
e	1
e^2	2
e^3	3

You're not required to know the decimal values of numbers like $1/e$ or e^2, but it can help.

Natural logarithms obey the same rules as logarithms using any base.

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^r) = r \cdot \ln(a)$$

Once again, it can help to describe those rules of logarithms using words.

Also, since they are inverses, exponential and logarithmic functions 'undo' each other:

$$\ln(e^w) = w \qquad \text{and} \qquad e^{\ln w} = w$$

You can describe this in words as well. The logarithm of the exponential of w, and the exponential of the logarithm of w, are both just w.