Rates of change

The average rate of change of the function f on the interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}$$

This can also be described as

$$\frac{\text{change in output}}{\text{change in input}} \qquad \text{or} \qquad \frac{\text{change in } y}{\text{change in } x}$$

EXAMPLE:
Suppose Dr. Mercer weighed 190 pounds on June 1st and weighed 180 pounds on July 1st. Find the average rate of change of his weight over that time period.

EXAMPLE:
A ball starts rolling down a ramp at noon. Its distance from the starting point (in feet) is

$$f(t) = t^2$$

where t is the number of seconds elapsed since noon.
Find the average rate of change of f over the interval $[3, 3.001]$.

(What if the interval was $[3, 3+h]$ where h is an arbitrary small number?)

The limit of a function

A function can be described using a formula or a graph.

EXAMPLE: Suppose $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 9}$. Notice that $f(3)$ is *undefined.*

Also notice that if x is 2.99 or 2.999 or 3.01 or 3.001, then $f(x)$ **IS** defined.

So it's meaningful to ask: What is the tendency when we look at

$$f(2.99),\ f(2.999),\ f(2.9999),\ \ldots \qquad \text{and/or} \qquad f(3.01),\ f(3.001),\ f(3.0001),\ \ldots\,?$$

$\lim\limits_{x\to 3} f(x)$ means: What does $f(x)$ do if x gets arbitrarily close to 3 (from *either* side)?

$\lim\limits_{x\to 3^+} f(x)$ means: What does $f(x)$ do if x is *greater* than 3 and gets arbitrarily close to 3?

$\lim\limits_{x\to 3^-} f(x)$ means: What does $f(x)$ do if x is *less* than 3 and gets arbitrarily close to 3?

QUESTION: Evaluate $\lim\limits_{x\to 3} \dfrac{x^2 - 5x + 6}{x^2 - 9}$.

(Can we rewrite $\dfrac{x^2 - 5x + 6}{x^2 - 9}$ somehow? Is there algebra we can do?)

Visualizing limits on a graph

The ideas behind one-sided and two-sided limits can be illustrated with a picture.

Suppose the graph of the function f is given below.

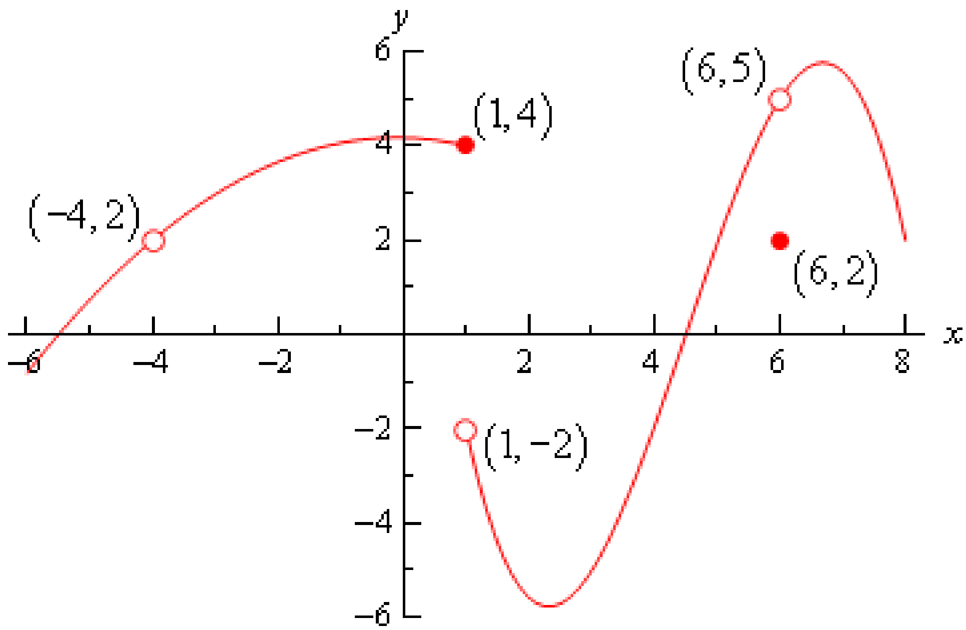

Image from Section 2.3 of Paul's Online Notes

$$f(-4) = \text{undefined} \qquad f(1) = 4 \qquad f(6) = 2$$

$$\lim_{x \to -4^-} f(x) = 2 \qquad \lim_{x \to 1^-} f(x) = 4 \qquad \lim_{x \to 6^-} f(x) = 5$$

$$\lim_{x \to -4^+} f(x) = 2 \qquad \lim_{x \to 1^+} f(x) = -2 \qquad \lim_{x \to 6^+} f(x) = 5$$

$$\lim_{x \to -4} f(x) = 2 \qquad \lim_{x \to 1} f(x) = \text{undefined} \qquad \lim_{x \to 6} f(x) = 5$$