Special limits

There are some facts about limits of special functions that you will need to use sometimes. These limit facts are easier to remember if you're familiar with those special functions.

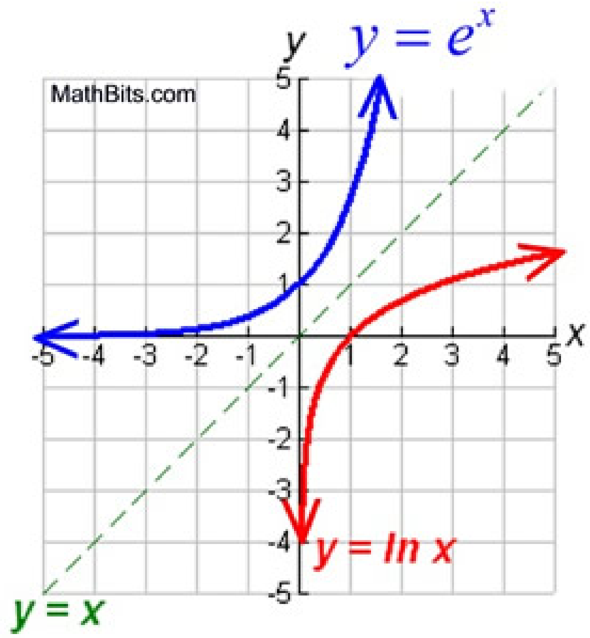

Image from MathBits.com

FACTS:

$$\lim_{x\to-\infty} e^x = 0 \qquad \lim_{x\to\infty} e^x = \infty \qquad \lim_{x\to 0^+} \ln x = -\infty \qquad \lim_{x\to\infty} \ln x = \infty$$

Very informally, we could paraphrase those limit facts as

$$e^{-\infty} = 0 \qquad e^{\infty} = \infty \qquad \ln 0 = -\infty \qquad \ln \infty = \infty$$

but ∞ and $-\infty$ are not *points* on the number line so this is technically wrong!

Remember how the inverse tangent function is formed from the regular tangent function.

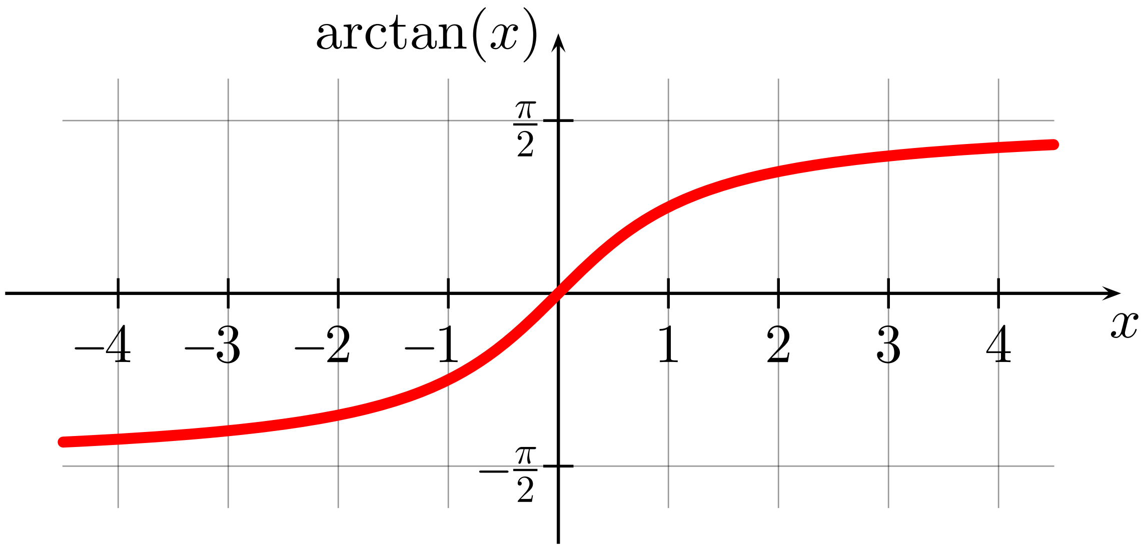

Image from Wikipedia

FACTS:

$$\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2} \qquad \lim_{x\to\infty} \arctan x = \frac{\pi}{2}$$

Very informally, $\arctan(-\infty) = -\frac{\pi}{2}$ and $\arctan(\infty) = \frac{\pi}{2}$, but this is technically wrong!

The Squeeze Theorem. Suppose $f(x) \leq g(x) \leq h(x)$, and suppose

$$\lim_{x\to c} f(x) = L \qquad \text{and} \qquad \lim_{x\to c} h(x) = L.$$

Then we must have $\lim_{x\to c} g(x) = L$ as well.

(If Gina is always between Fran and Holly, and Fran and Holly both approach the library, then Gina must approach the library as well.)

Small angle approximation for $\sin x$

FACT: We have $\lim_{x \to 0} \frac{\sin x}{x} = 1$. (Notice that x is approaching 0 here!)

Informally, this says that **IF** x is near 0, then $\frac{\sin x}{x} \approx 1$, or equivalently, $\sin x \approx x$.

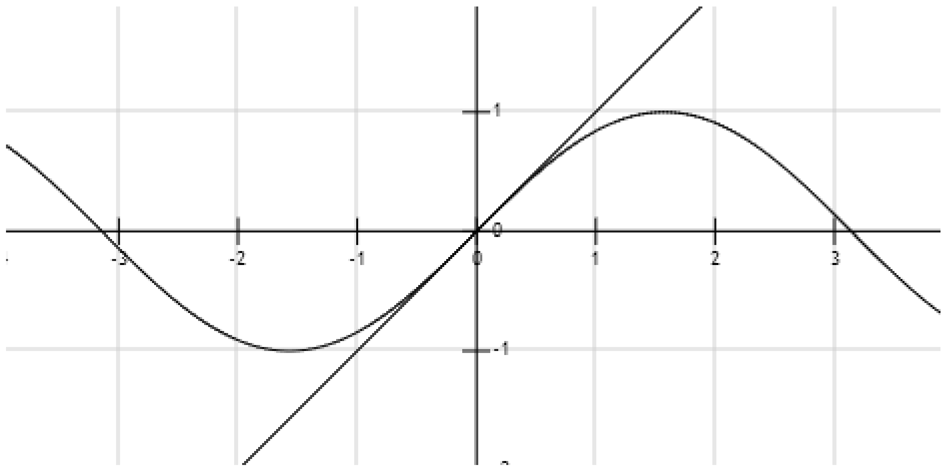

Image from `math.stackexchange.com`

IF x is near 0, then the line $y = x$ is close to the curve $y = \sin x$.

Continuity

A function $f(x)$ is **continuous** at $x = a$ if:

- $f(a)$ exists
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x)$ is equal to $f(a)$.

Polynomials, exponentials, sine, and cosine are continuous at all real numbers.

Types of discontinuities

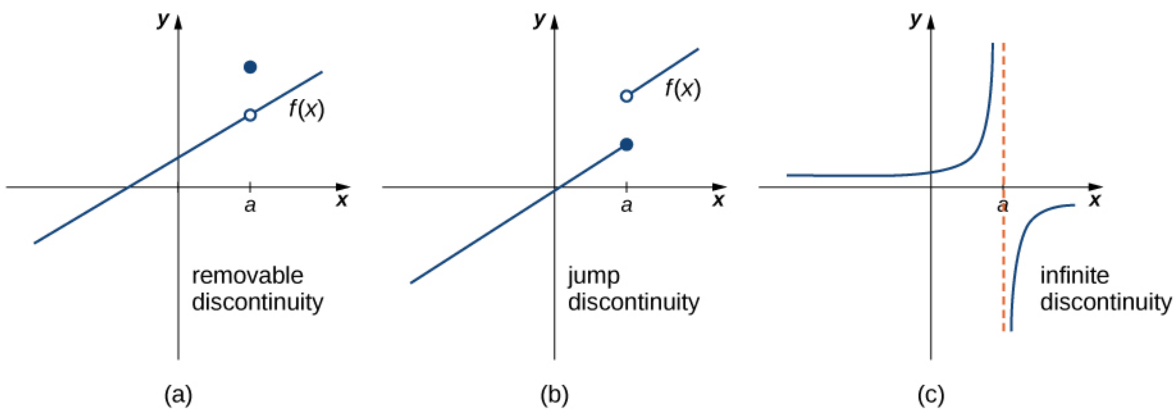

Image from Lumen Learning

Removable discontinuity (hole): $\lim_{x \to a} f(x)$ exists, but is not equal to $f(a)$

Jump discontinuity: $\lim_{x \to a^-} f(x)$ is different from $\lim_{x \to a^+} f(x)$

Infinite discontinuity: Either $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$ or both