MAC 2312
Idris Mercer, Spring 2020
Miscellaneous series practice
FIRST STEPS of solutions (and final answers)

Question 1. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n(n+1)}{(n+2)(n+3)}$$

GUESS: If n is big, then $\dfrac{n(n+1)}{(n+2)(n+3)} \approx \dfrac{n \cdot n}{n \cdot n} = 1 \not\to 0$

Test to use: nth term test for divergence
ANSWER: Diverges

Question 2. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1}{n+4}$$

GUESS: If n is big, then $\dfrac{1}{n+4} \approx \dfrac{1}{n}$ so series resembles harmonic series

Test to use: Direct comparison test (or limit comparison test)
ANSWER: Diverges

Question 3. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+3}$$

GUESS: If n is big, then $\dfrac{n}{n^2+3} \approx \dfrac{n}{n^2} = \dfrac{1}{n}$ so series resembles harmonic series

Test to use: Limit comparison test (or integral test)
ANSWER: Diverges

Question 4. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} e^{-2n}$$

GUESS: Notice $e^{-2n} = \dfrac{1}{e^{2n}} = \left(\dfrac{1}{e^2}\right)^n$

Test to use: Recognize series as geometric series
ANSWER: Converges

Question 5. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2}{10^n}$$

GUESS: Notice $\dfrac{2}{10^n} = 2 \cdot \left(\dfrac{1}{10}\right)^n$

Test to use: Recognize series as geometric series
ANSWER: Converges

Question 6. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{n!}{1000^n}$$

GUESS: Use guideline that factorials grow faster than exponentials

Test to use: Ratio test
ANSWER: Diverges

Question 7. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$$

GUESS: If n is big, then $\dfrac{2^n + 4^n}{3^n + 4^n} \approx \dfrac{4^n}{4^n} = 1 \not\to 0$

Test to use: nth term test for divergence
ANSWER: Diverges

Question 8. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n}$$

GUESS: Use guideline that $\ln n$ is bigger than a constant but smaller than any power of n

Test to use: Direct comparison test
ANSWER: Diverges

Question 9. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2}{e^n}$$

GUESS: Use guideline that exponentials grow faster than polynomials

Test to use: Ratio test
ANSWER: Converges

Question 10. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{7}{\sqrt{n+4}}$$

GUESS: If n is big, then $\dfrac{7}{\sqrt{n+4}} \approx \dfrac{7}{\sqrt{n}}$

Test to use: Limit comparison test
ANSWER: Diverges

Question 11. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{5n+10\sqrt{n}}$$

GUESS: If n is big, then $\dfrac{1}{5n+10\sqrt{n}} \approx \dfrac{1}{5n}$

Test to use: Limit comparison test
ANSWER: Diverges

Question 12. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$$

GUESS: Use guideline that $\ln n$ is bigger than a constant but smaller than any power of n

Test to use: Direct comparison test
ANSWER: Diverges

Question 13. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$$

GUESS: If n is big, then $\dfrac{5^n}{4^n + 3} \approx \dfrac{5^n}{4^n} = \left(\dfrac{5}{4}\right)^n \not\to 0$

Test to use: nth term test for divergence
ANSWER: Diverges

Question 14. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2^n}{n + 1}$$

GUESS: Use guideline that exponentials grow faster than polynomials

Test to use: nth term test for divergence
ANSWER: Diverges

Question 15. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$$

GUESS: Use guideline that $\ln n$ grows more slowly than any power of n

Test to use: nth term test for divergence
ANSWER: Diverges

Question 16. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$$

GUESS: Use fact that $\cos^2 n = (\cos n)^2$ can be bounded between 0 and 1

Test to use: Direct comparison test
ANSWER: Converges

Question 17. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n3^n}$$

GUESS: Use fact that $n3^n$ grows faster than 3^n

Test to use: Direct comparison test (or ratio test)
ANSWER: Converges

Question 18. Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

GUESS: Use guideline that $\ln n$ grows more slowly than any power of n

Test to use: Direct comparison test
ANSWER: Converges

Question 19. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n!}{n^2}$$

GUESS: Use guideline that $n!$ grows faster than n^2

Test to use: Ratio test
ANSWER: Diverges

Question 20. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2^n}{n3^n}$$

GUESS: Use fact that $n3^n$ grows faster than 3^n

Test to use: Direct comparison test (or ratio test)
ANSWER: Converges

Question 21. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^{1.4}}{2^n}$$

GUESS: Use guideline that exponentials grow faster than power functions

Test to use: Ratio test
ANSWER: Converges

Question 22. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$$

GUESS: Use guideline that exponentials grow faster than power functions

Test to use: Ratio test
ANSWER: Converges

Question 23. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.25^n}$$

GUESS: Use fact that $(-1)^n = \pm 1$, so $2 + (-1)^n$ is bounded

Test to use: Direct comparison test
ANSWER: Converges

Question 24. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

GUESS: We see factorials, so try ratio test

Test to use: Ratio test
ANSWER: Converges

Question 25. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$$

GUESS: Notice it looks like an alternating p-series with $p \leq 1$

Test to use: Alternating series test, **and** facts about p-series
ANSWER: Converges conditionally

Question 26. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$$

GUESS: Notice it looks like an alternating p-series with $p > 1$

Test to use: Alternating series test, **and** facts about p-series
ANSWER: Converges absolutely

Question 27. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n3^n}$$

GUESS: Notice that $n3^n$ grows even faster than 3^n

Test to use: Ratio test
ANSWER: Converges absolutely

Question 28. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=2}^{\infty}(-1)^n \frac{4}{(\ln n)^2}$$

GUESS: Use guideline that $\ln n$ grows more slowly than any power of n

Test to use: Alternating series test, **and** direct comparison with p-series
ANSWER: Converges conditionally

Question 29. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$$

GUESS: If n is big, then $\dfrac{n}{n^2+1} \approx \dfrac{n}{n^2} = \dfrac{1}{n}$, so series resembles alternating harmonic series

Test to use: Alternating series test, **and** limit comparison with p-series
ANSWER: Converges conditionally

Question 30. Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2+5}{n^2+4}$$

GUESS: If n is big, then $\dfrac{n^2+5}{n^2+4} \approx \dfrac{n^2}{n^2} = 1 \not\to 0$

Test to use: nth term test for divergence
ANSWER: Diverges