Does Data Have a Place in a Calculus Course?

Where are our students going?

Jobs:

Health care and data science

Deb Hughes Hallett
University of Arizona/Harvard Kennedy School

https://www.hoffensoft.com/ar
https://www.ringlead.com

“The rising demand for analytics in the companies has given rise to an exponential shortage of data scientists”

https://analyticsindiamag.com/reasons-why-there-is-a-shortage-of-data-scientists-in-the-industry/

“Data is the new oil.”
Geoffrey Moore
“Without big data, you are blind and deaf and in the middle of a freeway.”
Clive Humby
“Data will talk to you if you are willing to listen”
Jim Bergeson

Teaching with Data

Issues

- **Data is generally discrete:**
  - Make assumptions
  - Approximate
- **Data is messy---not "perfect":**
  - Smoothing
  - Approximate
- **Mathematical questions:**
  - Role of parameters
  - Making projections
  - Finding model rather than given the model
- **Tools:**
  - Spreadsheets or calculators
  - Scatterplots

Where Data Fits

- **Covid-19 data**
  - **Families of functions:**
    - **Linear, exponential**
    - **Logarithms (and log scales)**
    - **Role of parameters**
  - **Differentiation:**
    - **Graphical**
  - **Integration**
    - **Graphical**
  - **Differential equations**
- **Climate change**
  - **Families of functions:**
    - **Linear, exponential**
    - **Role of parameters**
  - **Integration:**
    - **Improper integrals**

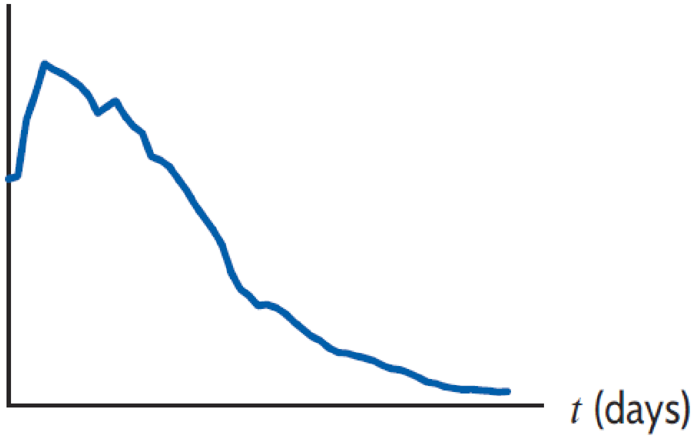


Figure 3

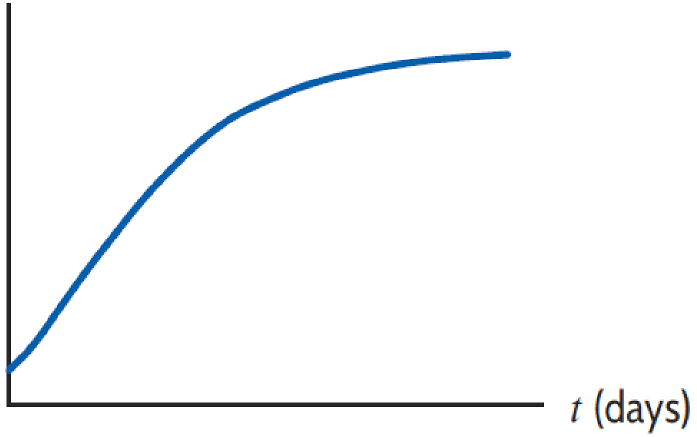


Figure 4

COVID CASES
Switzerland: Spring 2020

What data are we looking at?

- $P(t)$ = Total cases confirmed up to and including day t
- $N(t)$ = Newly confirmed cases on day t
- t is time in days

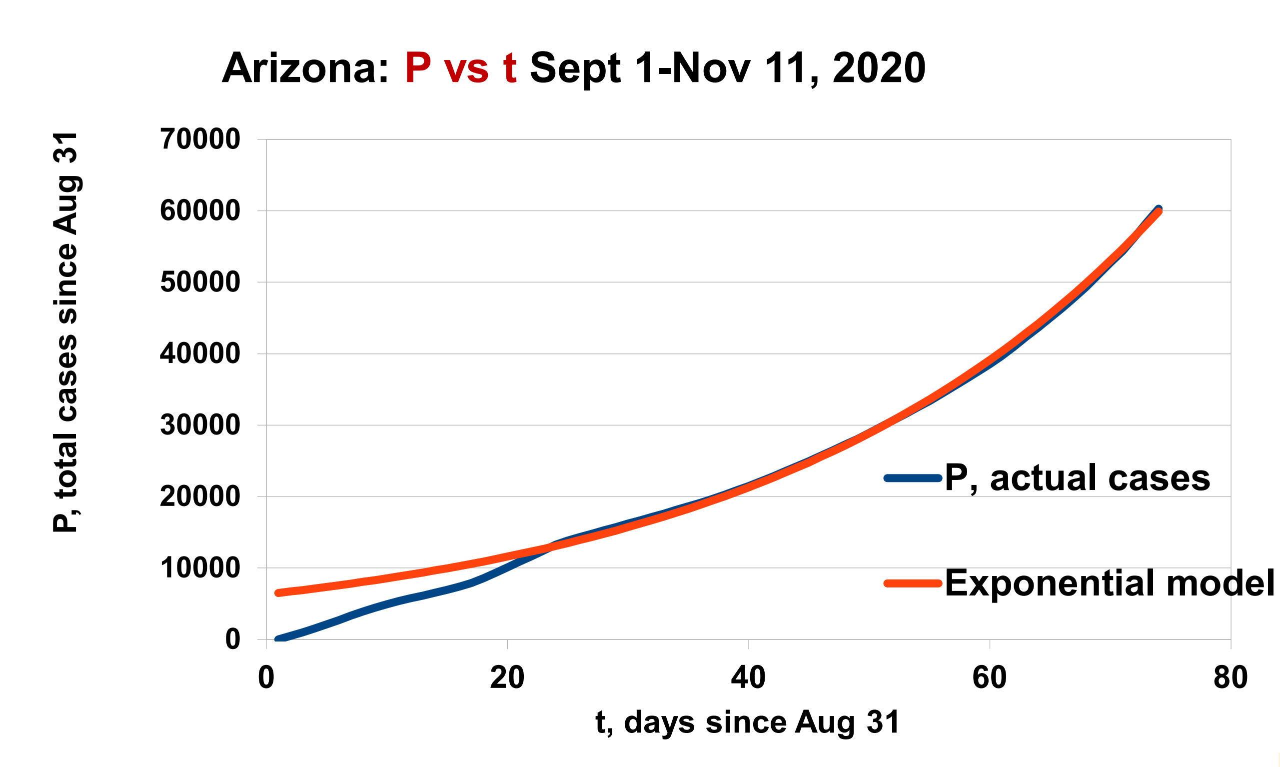
Questions

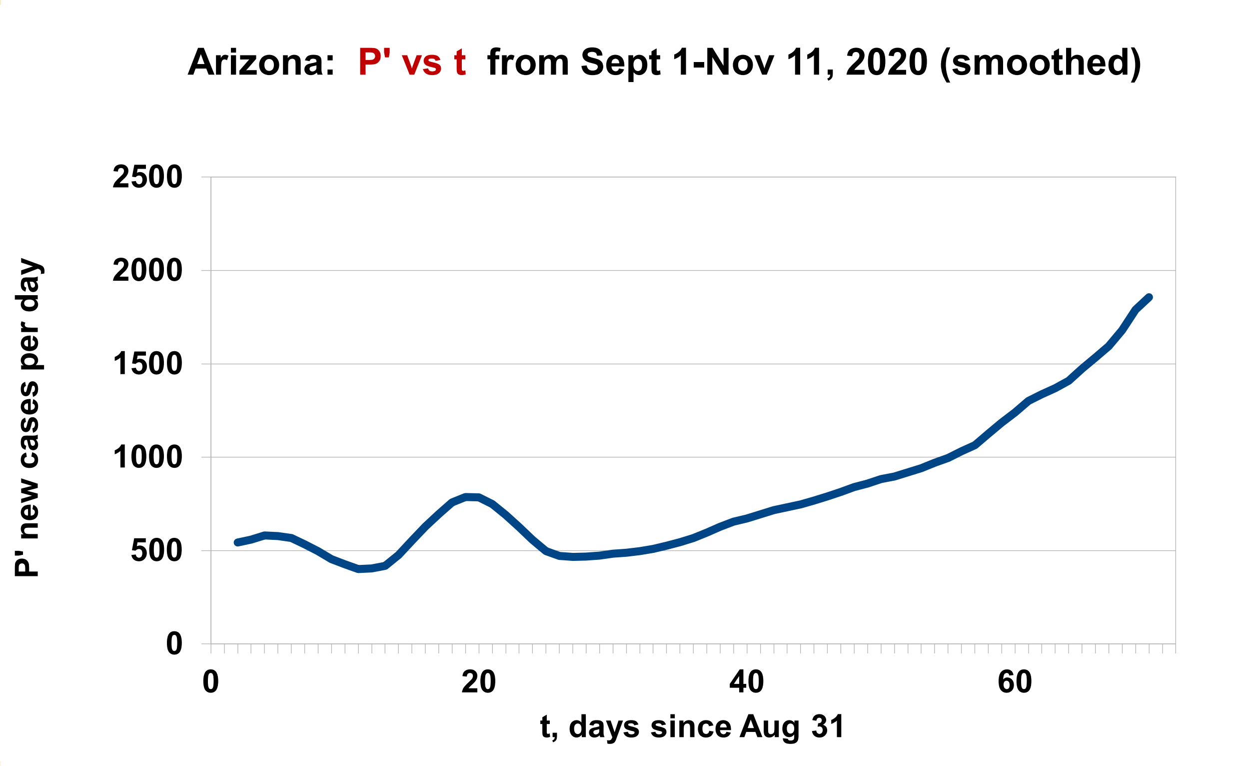
1. **Which graph is P and which is N?**
    - Upper graph is N
    - P can't decrease
2. **What is the (approximate) relationship between P and N?**
    - With $\Delta t = 1$, we see $\frac{dP}{dt} \approx N$

https://mcwg.github.io/covid/
https://isaac-flath.shinyapps.io/coronavirus2/

How Do We Find a Model?

P is second wave Cumulative Cases





If exponential model fits, what should daily cases look like?

Supports exponential model but it is not sufficient

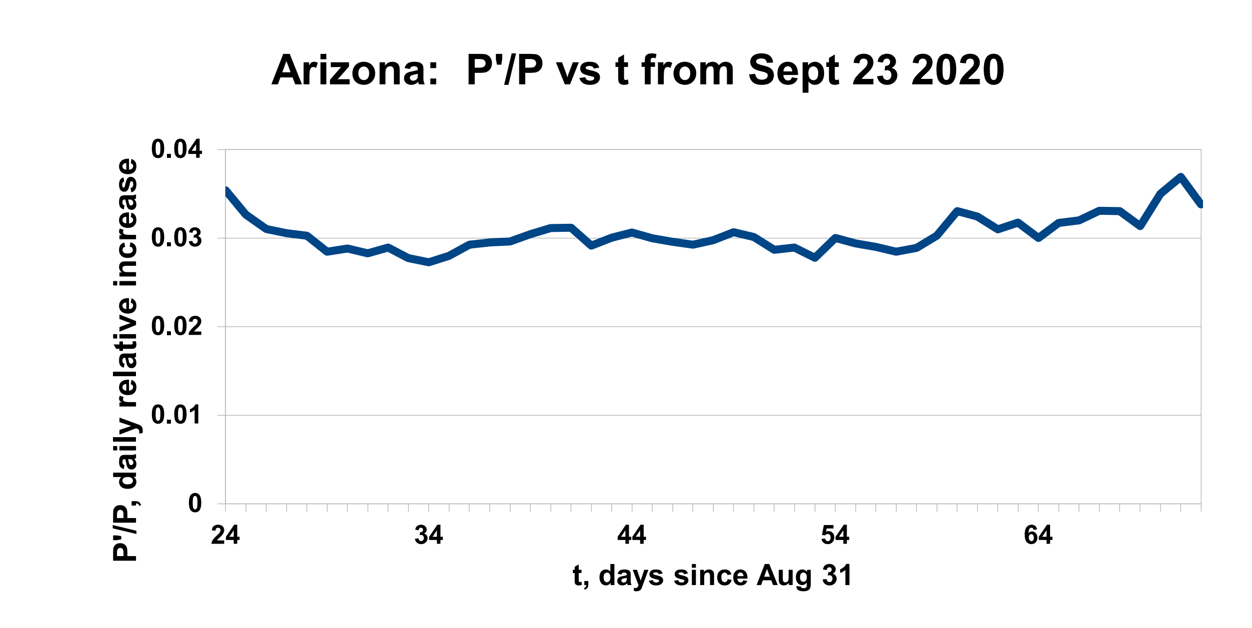
Derivative P' is mostly increasing and concave up, but is P exponential?

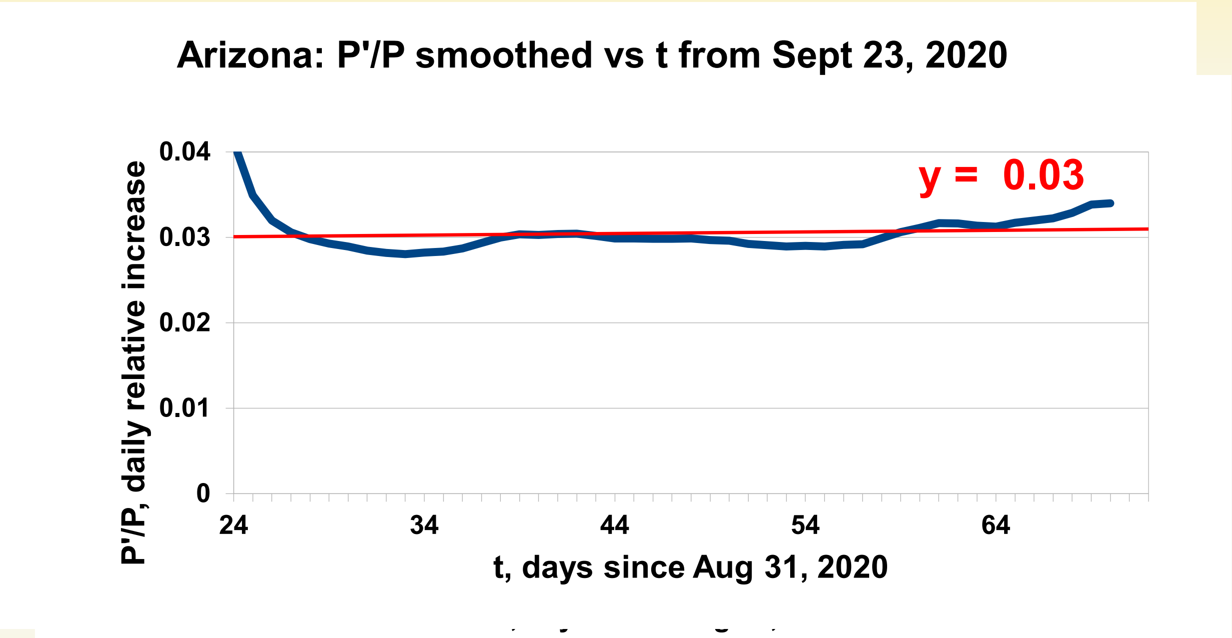
Finding a Model: P is Cumulative Cases

Scatter Plot: P'/P versus t

Arizona

t from Sept 1, 2020





Why is graph a horizontal line? What does this tell us?

Exponential Growth at 3% continuous per day

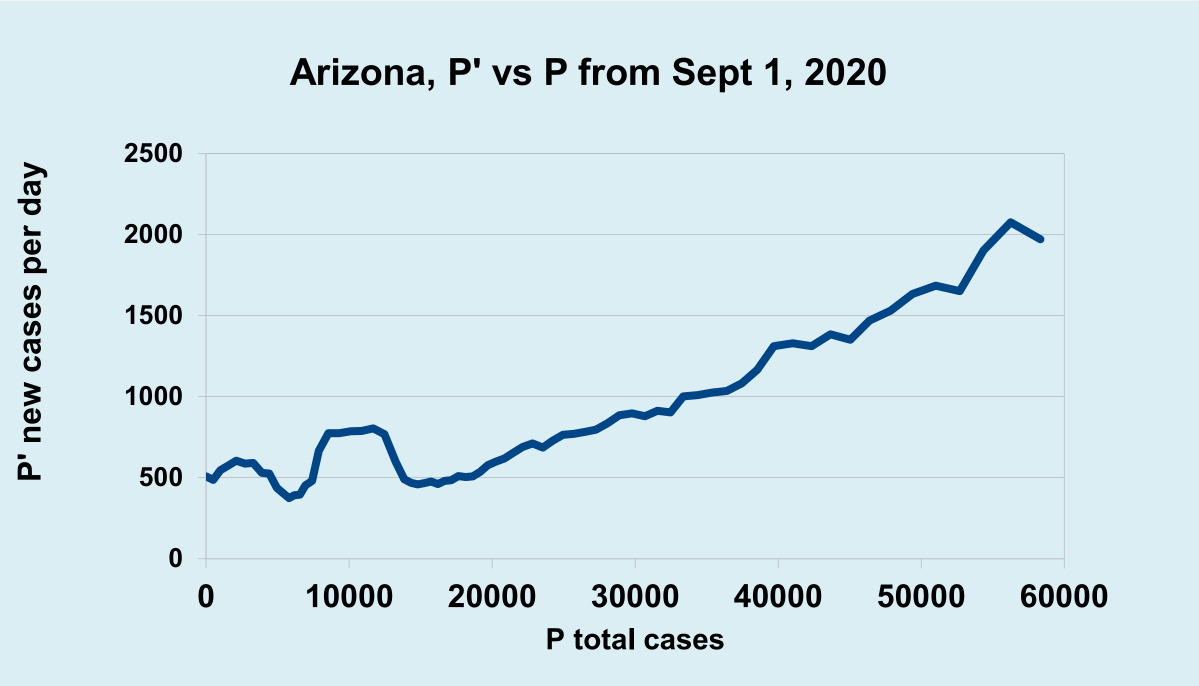
$$P = 6,850e^{0.03t}$$

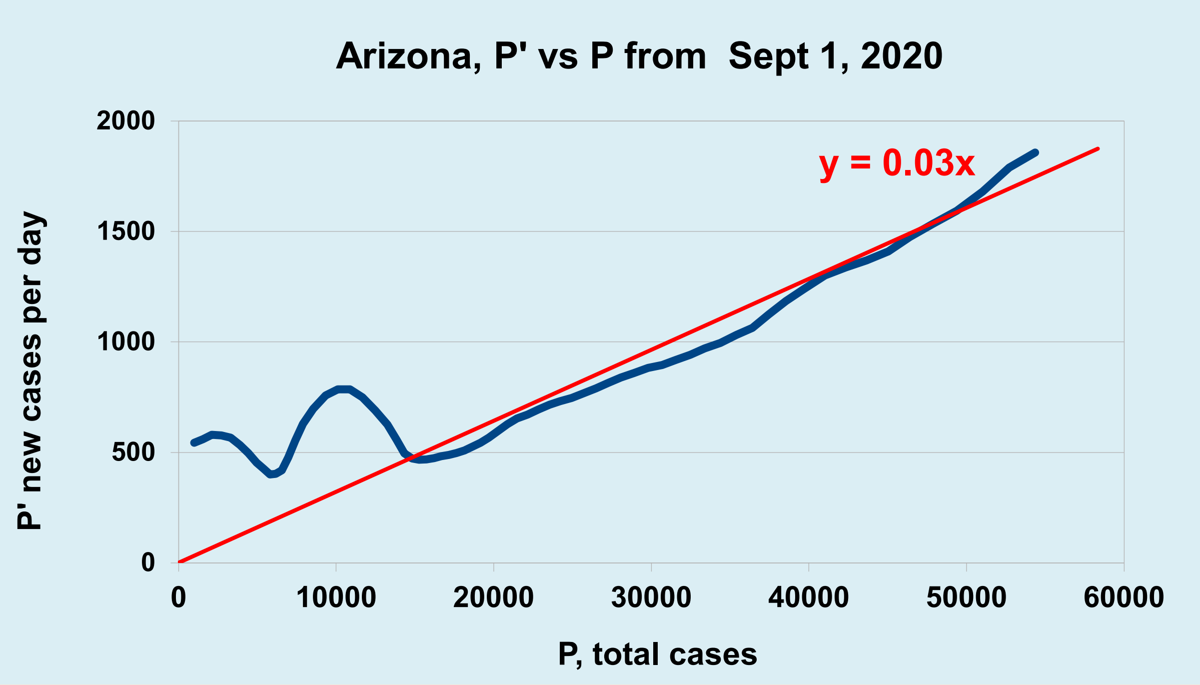
Equation: $\frac{P'}{P} = 0.03$

$$P' = 0.03P$$

Scatter plot: P' versus P
Phase Planes and Trajectories

Arizona
t from Sept 1, 2020





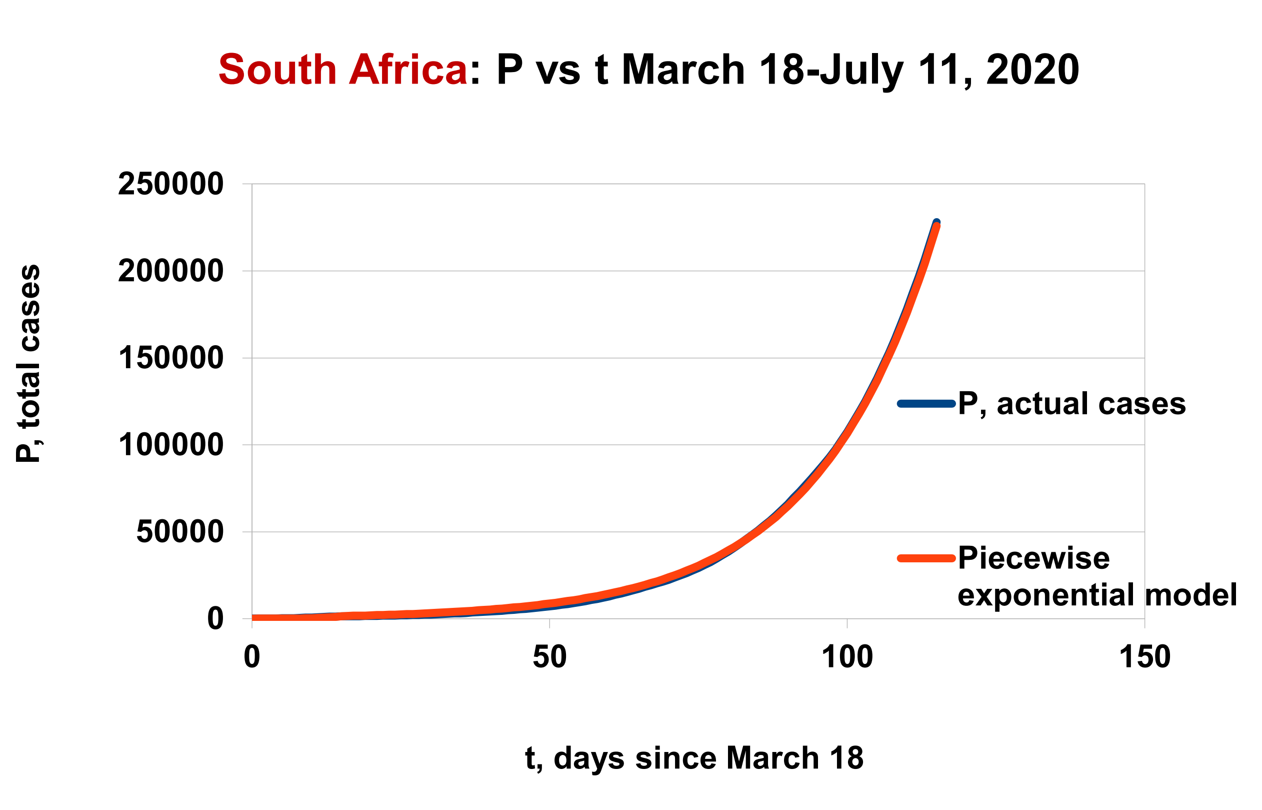
The graph a line through the origin: What does it tell us?

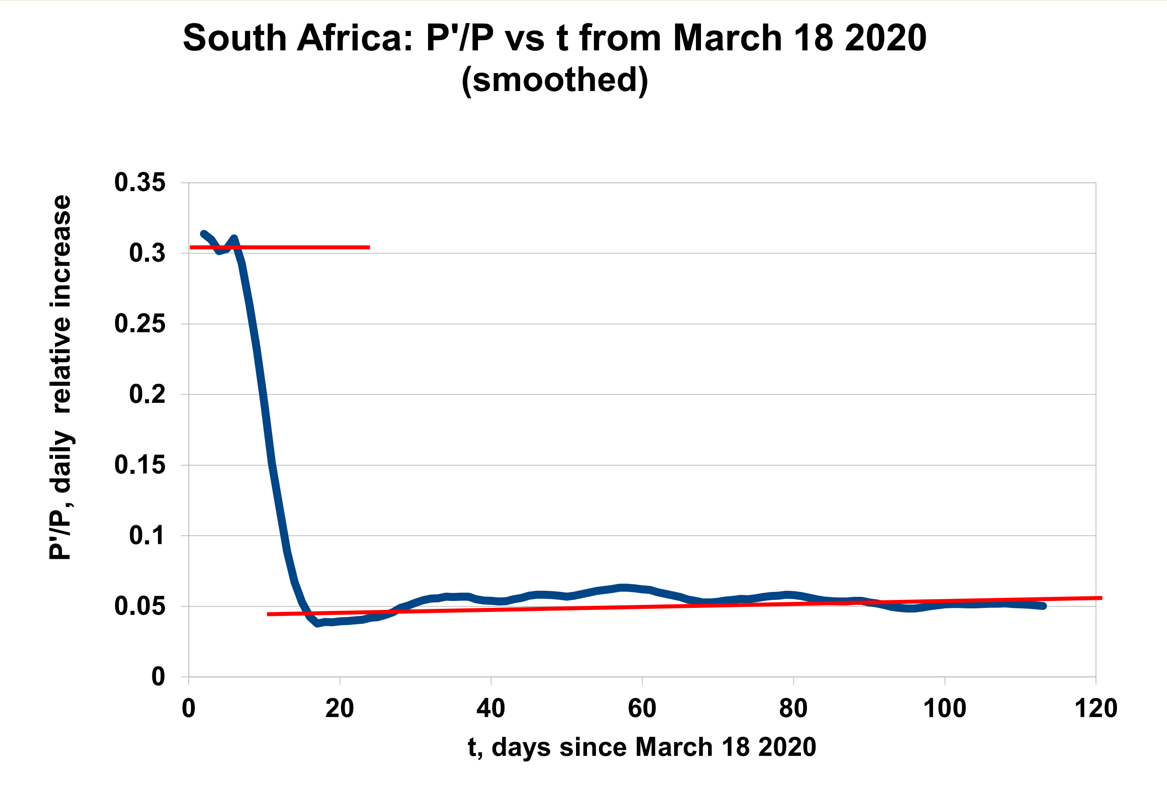
Exponential:
Line though origin, slope k, the growth rate

Slope = 0.03 is exponential growth rate

$$P' = 0.03P$$

Exponential Models Up Close

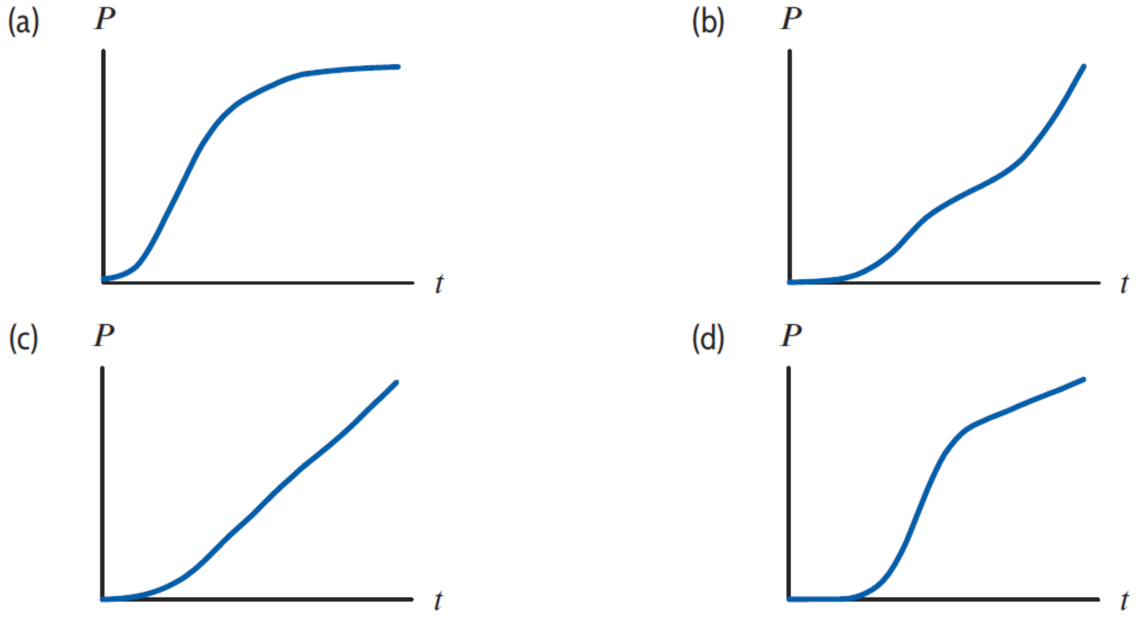




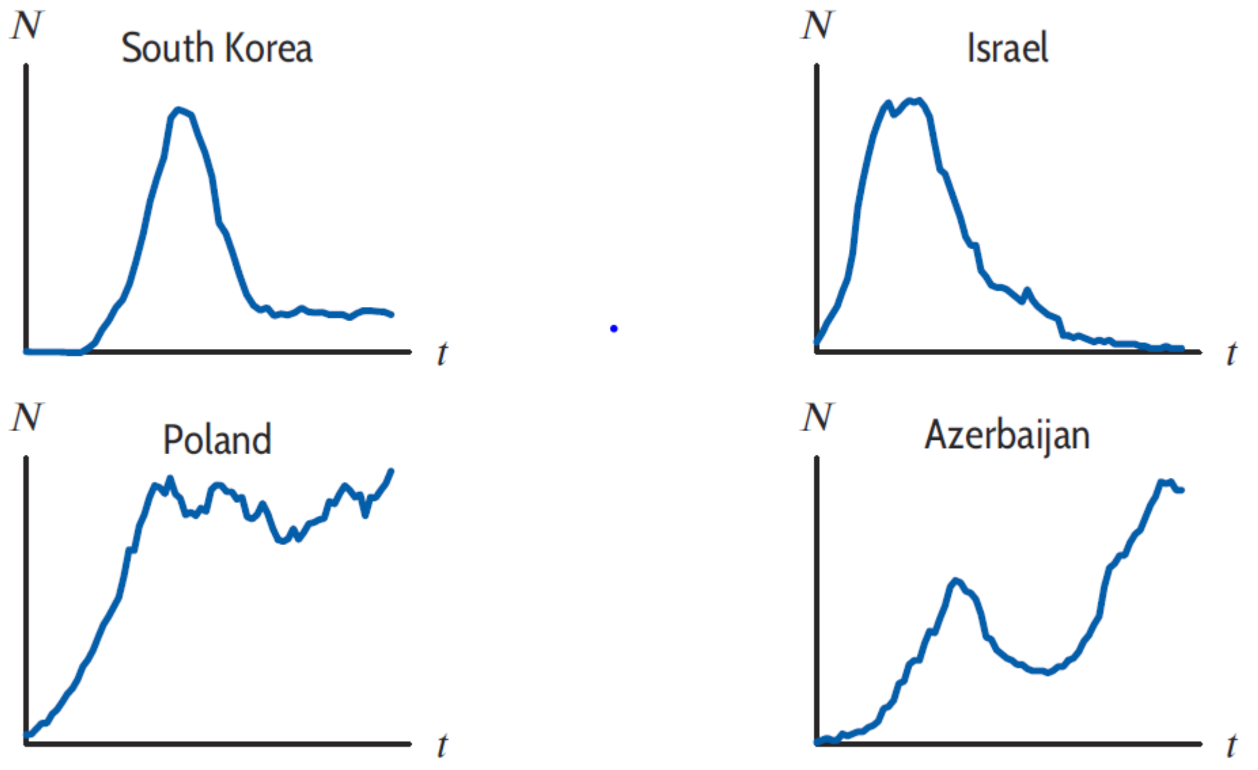
Exponential Growth at 30% and 5% continuous per day. After April 4

$$P = 1600e^{0.05t}$$

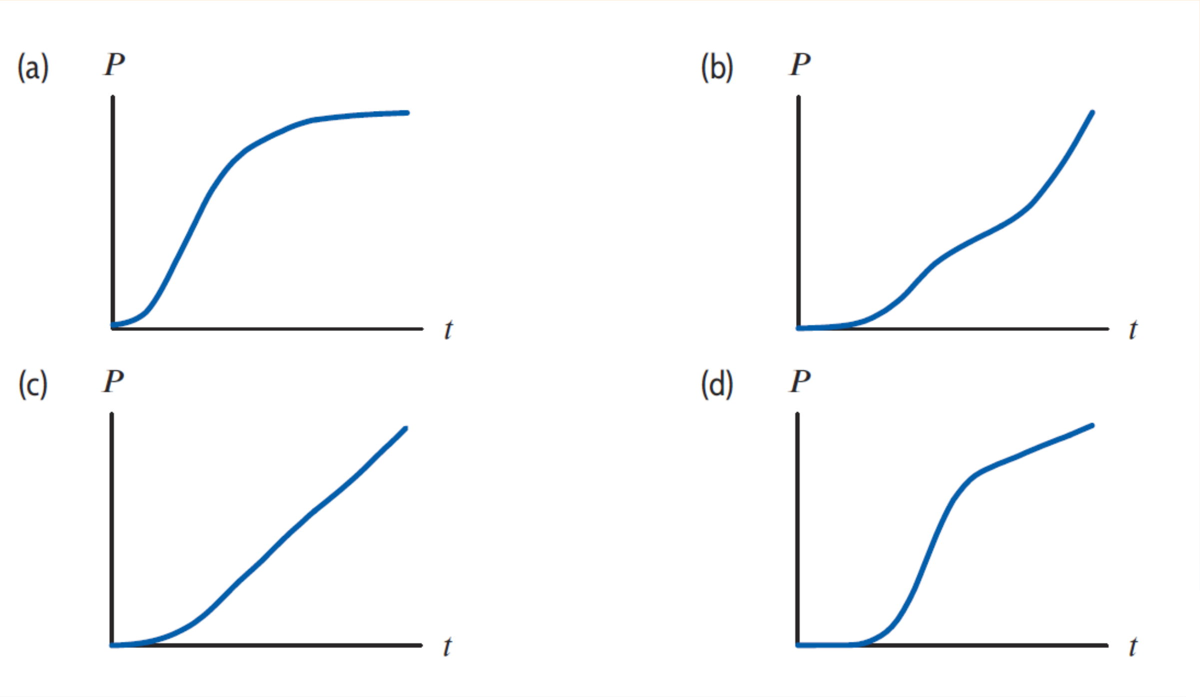
On March 23, 2020 ($t = 5$), daily increase in total cases was $0.3 = 30\%$. That day, South Africa announced a lockdown and ten days later it was $0.05 = 5\%$.

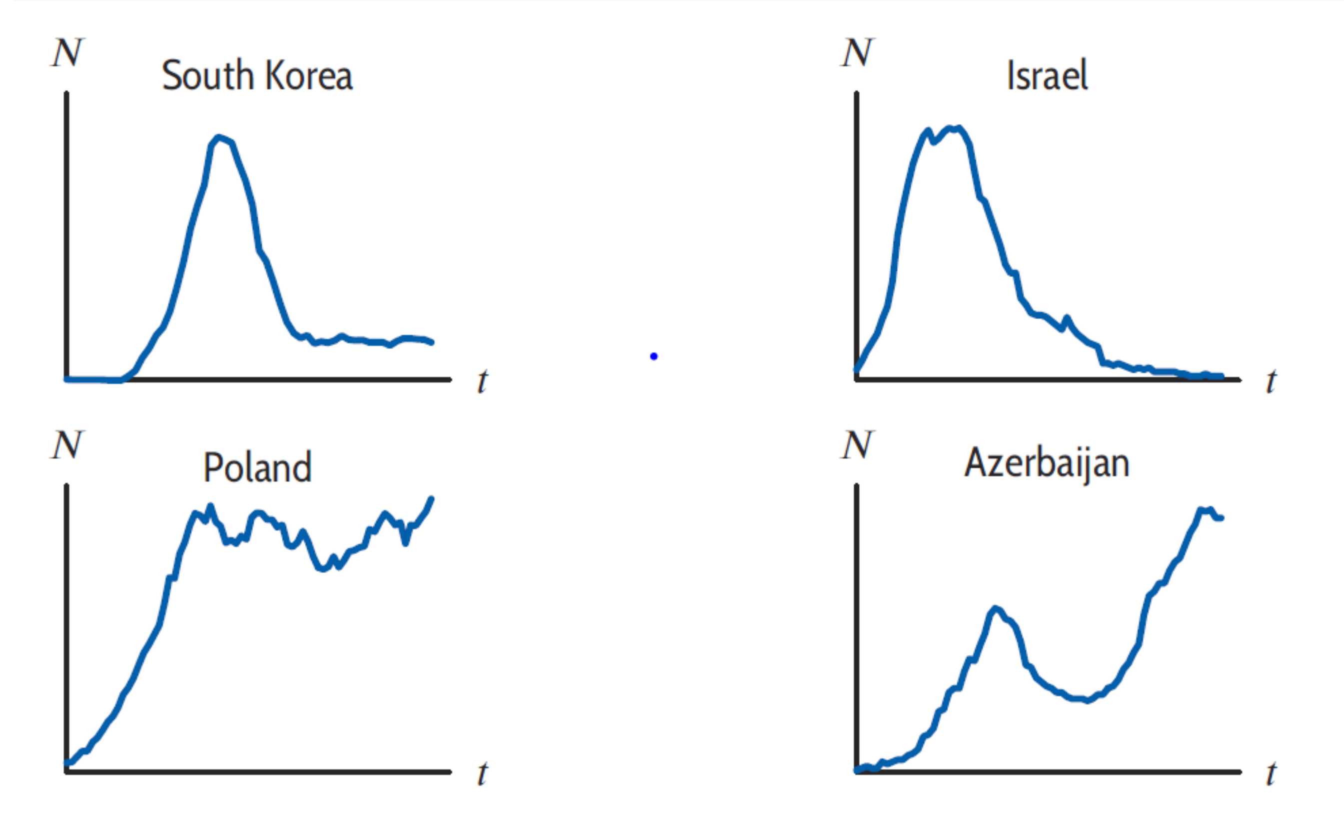


Which P graph corresponds to which N graph?



Which P graph corresponds to which N graph?



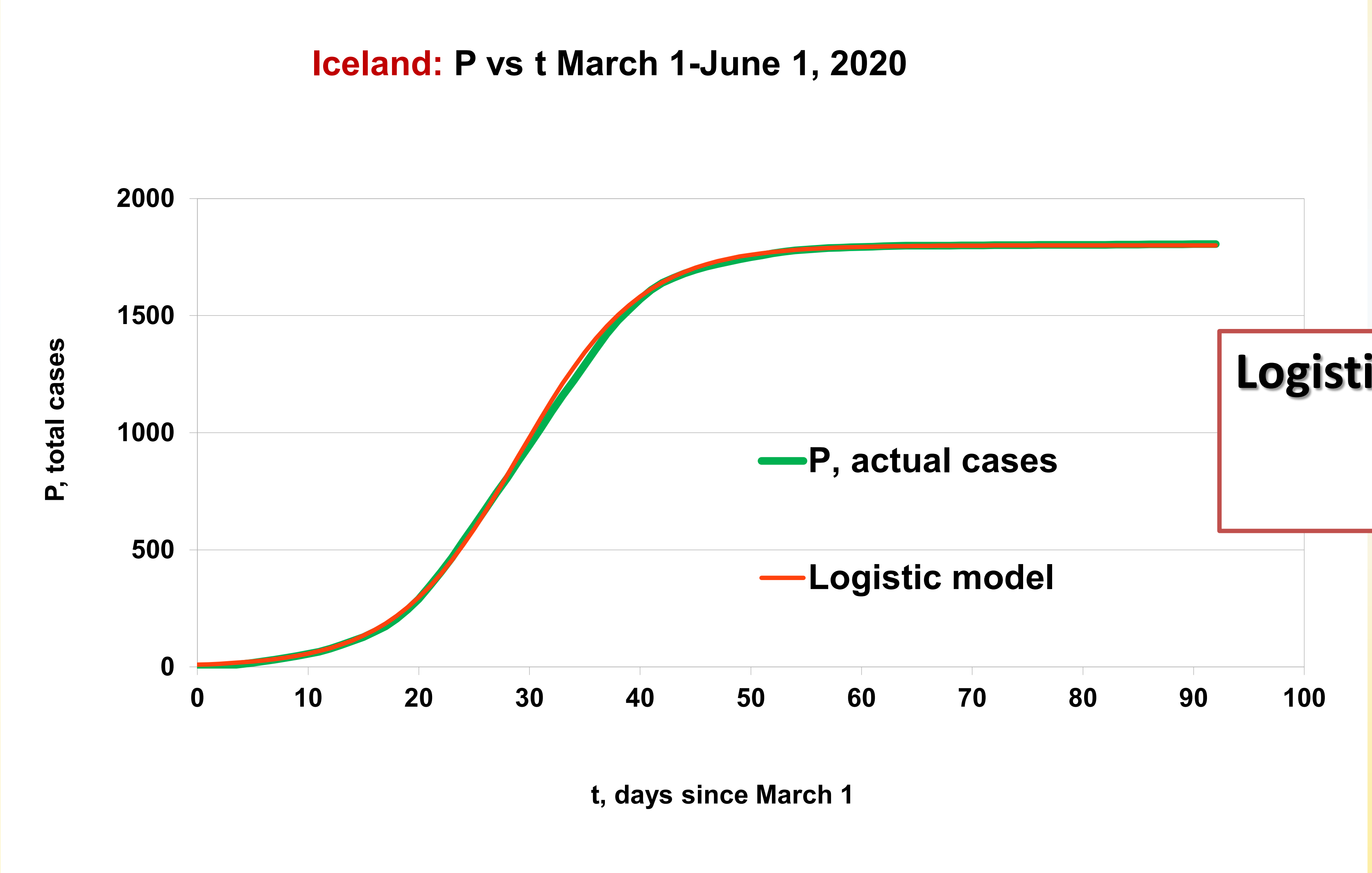


(a) Is Israel
(b) Is Azerbaijan
(c) Is Poland
(d) Is South Korea

Peaks of N $\leftrightarrow$ Steep part of P
Zeros of N $\leftrightarrow$ Flat part of P
Horizontal part of N $\leftrightarrow$ Straight part of P

How Do We Find a Model:

P is Cumulative Cases



Logistic Model:

$$P = \frac{1800}{1 + 185e^{-0.18t}}$$

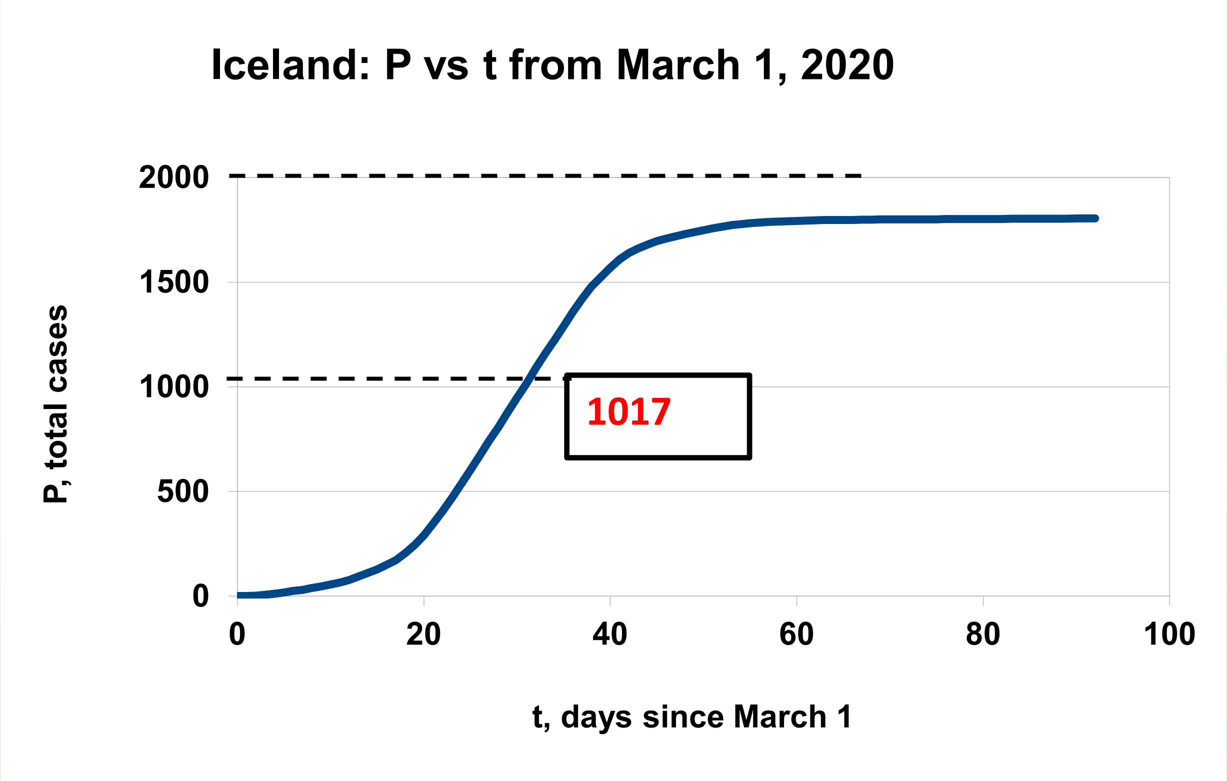
Logistic: Estimating Carrying Capacity L Early from P'

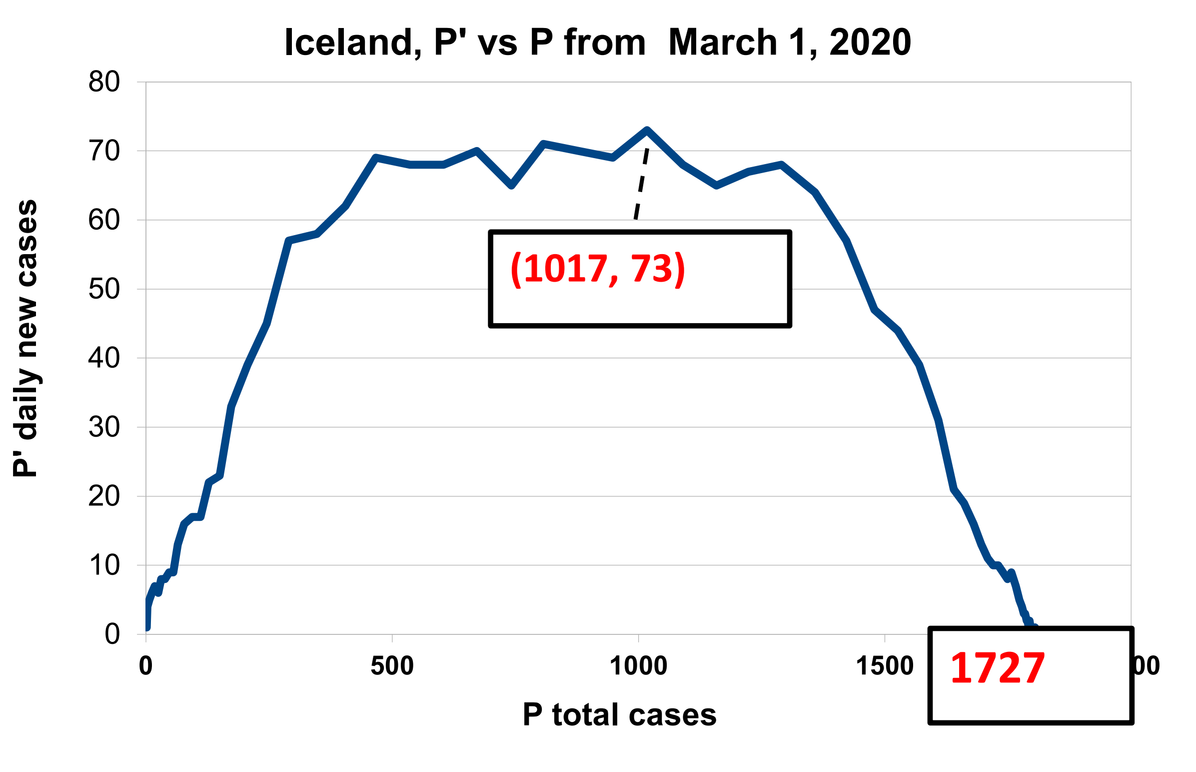
- **Logistic**: Carrying capacity, L
  - For $0 \leq P \leq L$, **maximum slope, P', occurs at $P = L/2$**
- So we conclude that carrying capacity is $\boldsymbol{L = 2P}$

- **Optimization Problem with P':**
  - Find P'' by differentiating $P' = \frac{k}{L}P(L - P)$ with respect to t, using implicit differentiation
  - Set $P'' = 0$ and solve for P getting $P = L/2$
  - Check critical point is a maximum

Estimating Logistic Parameter L Early from P'

- For $0 \le P \le L$, **maximum P' occurs at $P = L/2$**
- So we conclude $\boldsymbol{L = 2P}$
- Iceland: $2 \cdot 1017 = 2034$





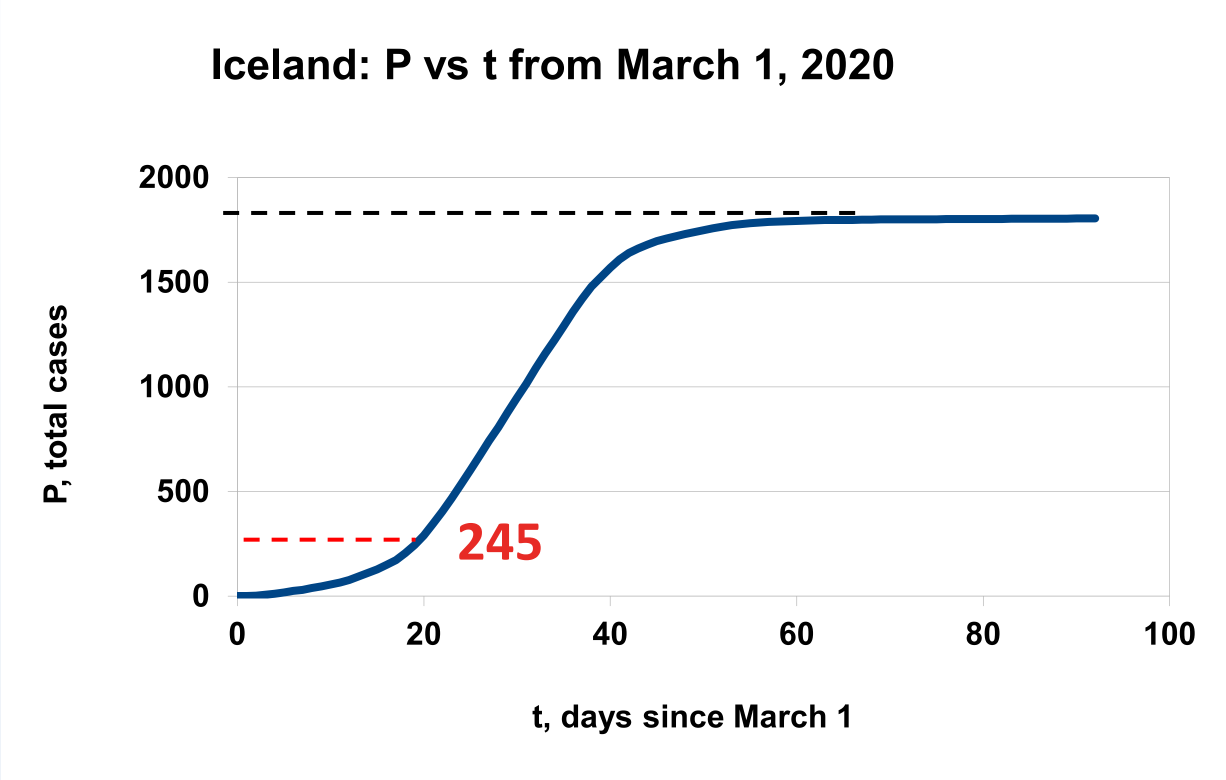
Logistic: Estimating Carrying Capacity L Earlier from P''

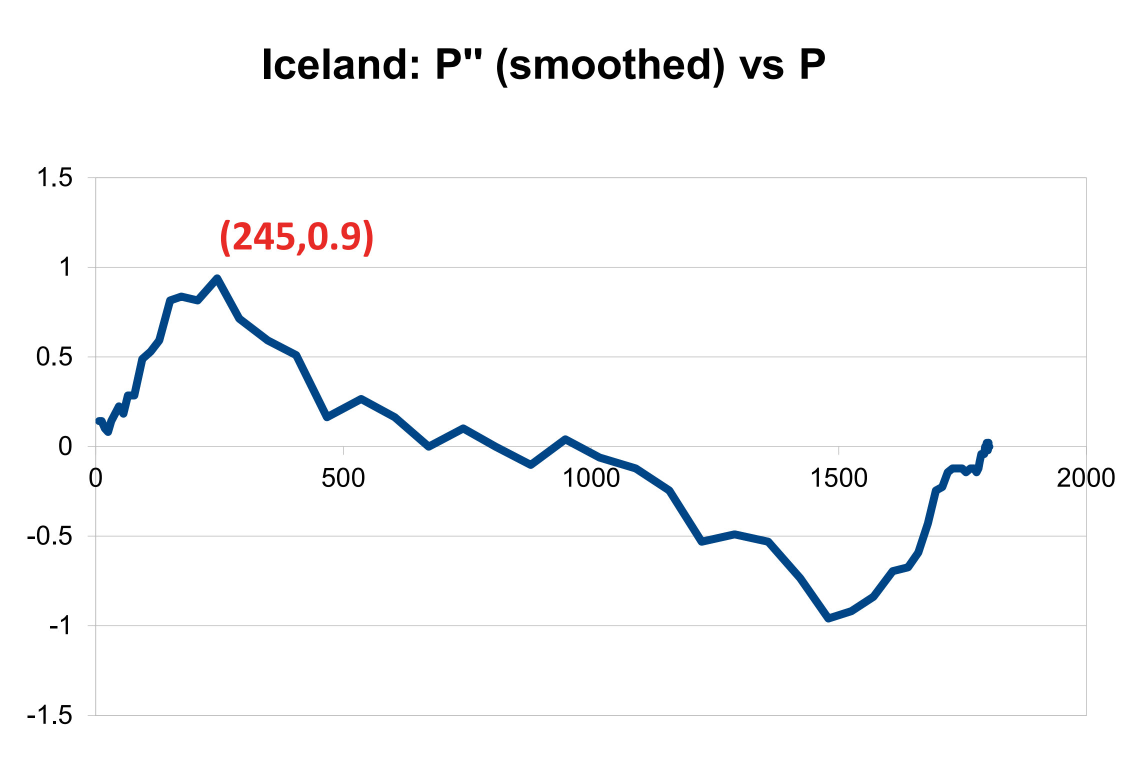
- **Logistic**: Carrying capacity, L
  - For $0 \leq P \leq L$, **maximum P'' occurs at $P = 0.211 \cdot L$**
- So we conclude carrying capacity $\boldsymbol{L = 4.7P}$

- **Optimization Problem with P''**
  - Find P'' by differentiating $P' = \frac{k}{L}P(L - P)$ with respect to t
  - Substitute for P' to get P'' in terms of P
  - Find P''' and set $P''' = 0$
  - Solve the quadratic for P for two critical points. Smaller gives the max

Estimating Logistic Parameter L Early from P''

- For $0 \leq P \leq L$, **maximum P'' occurs at $P = L/0.211$**
- So we conclude $\boldsymbol{L = 4.7P}$
- **Iceland: $4.7 \cdot 245 = 1151.5$**





Sea Level Rise and its Costs

Miami

Miami

Data

- Hurricane Sandy in New York caused up to $70 billion in damages
- Katrina caused $125 billion in losses in New Orleans, much of it due to rising water.
- Here we look at a simplified estimate of some of the threats facing Boston in the 21st C

New Jersey

Netherlands

https://www.miamitodaynews.com/2016/09/13/miami-getting-serious-sea-level-rise/
https://www.miamiherald.com/opinion/editorials/article224405100.html
http://www.bu.edu/articles/2012/lessons-from-hurricane-sandy/
https://www.climatecentral.org/news/floods-may-cost-coastal-cities-60-billion-annually-by-2050-16356

Future Flooding Predictions: Miami and Surroundings

https://coastal.climatecentral.org/map/13

COASTAL RISK SCREENING TOOL

LAND PROJECTED TO BE BELOW ANNUAL FLOOD LEVEL IN 2100

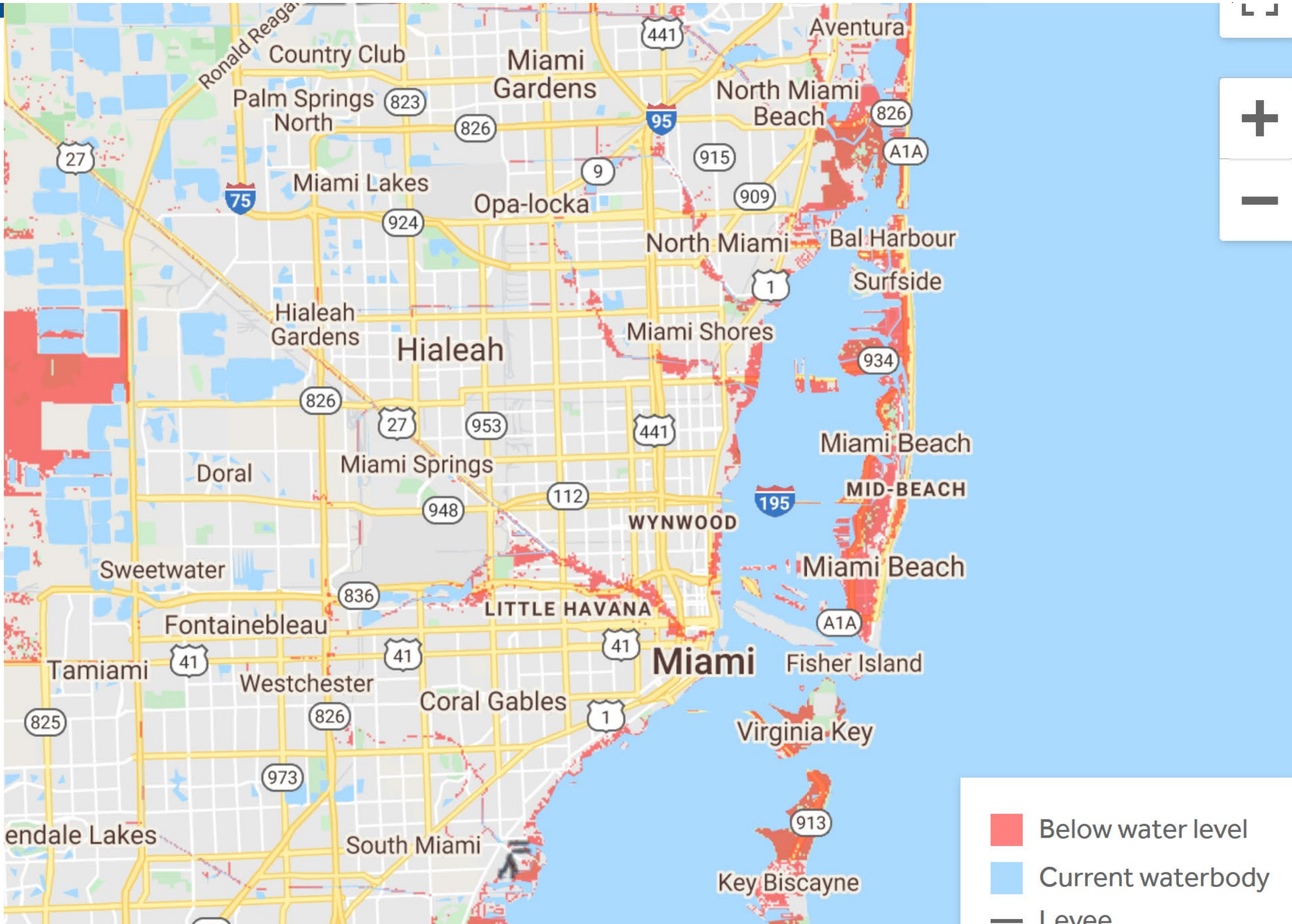
Explore sea level rise and coastal flood threats by adjusting the controls below.

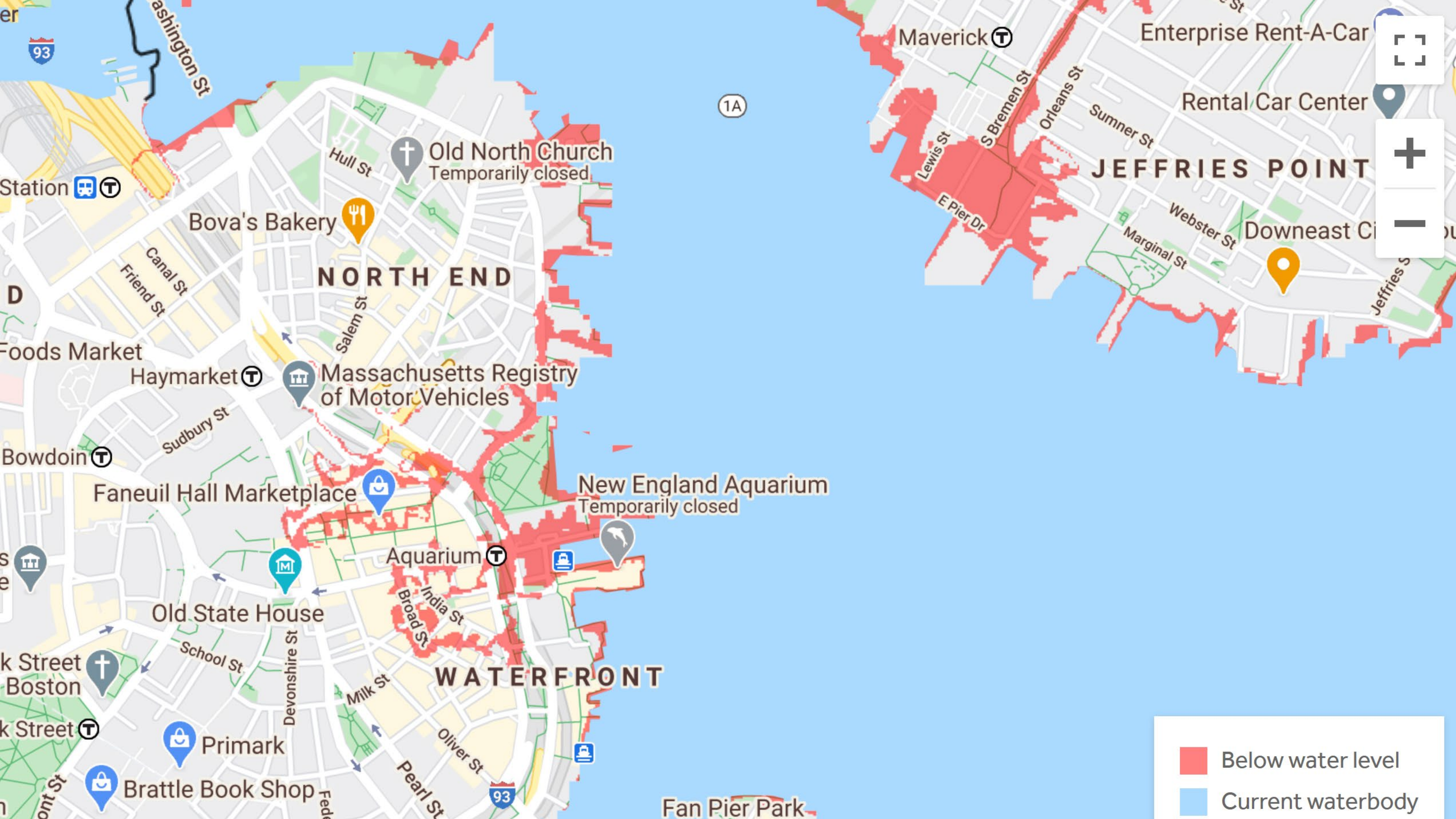
DETAILS AND LIMITATIONS

YEAR

2100

CHANGE OTHER SETTINGS




Old North Church
Temporarily closed
Bova's Bakery
NORTH END
Haymarket
Massachusetts Registry of Motor Vehicles
Bowdoin
Faneuil Hall Marketplace
New England Aquarium
Temporarily closed
Aquarium
Old State House
WATERFRONT
Primark
Brattle Book Shop
Fan Pier Park
Maverick
Enterprise Rent-A-Car
Rental Car Center
JEFFRIES POINT
Hull St
Salem St
Canal St
Friend St
Sudbury St
School St
Devonshire St
Milk St
India St
Broad St
Oliver St
Pearl St
Lewis St
S Bremen St
Orleans St
Sumner St
Webster St
Marginal St
E Pier Dr
Below water level
Current waterbody

https://e360.yale.edu/features/who-will-pay-for-the-huge-costs-of-holding-back-rising-seas

How Might We Address Future Damages?

"By 2070, some neighborhoods in Boston could flood monthly, resulting in $1.4 billion in annual losses to businesses and property."

(Climate Ready Boston https://www.boston.gov/departments/environment/preparing-climate-change)

How much should Boston be willing to spend in 2020 to avoid one of the $1.4 billion costs?

We imagine a choice between

- Investing $\$P$ billion today say at a 1% per year continuous interest rate
- Spending that money preventatively, to build sea walls and drainage channels

First Option

To meet the $1.4 bn payment, $\$P$ bn should grow to $1.4 bn 50 years, so in billions

$$P = 1.4e^{-0.01 \cdot 50} = 0.849$$

That is, **about $849 million**.

Budget comparison: For 2021, Boston's operating budget is $3.6 billion, capital budget is $3.0 billion for 2021-2025.

- $1,400 mil is **future value**, ***FV***
- $849 mil is **present value** , ***PV***
- 1% per year is the **discount rate**

$$\boldsymbol{PV = FVe^{-0.01t}}$$

https://www.boston.gov/departments/budget/

Time Value of Payments

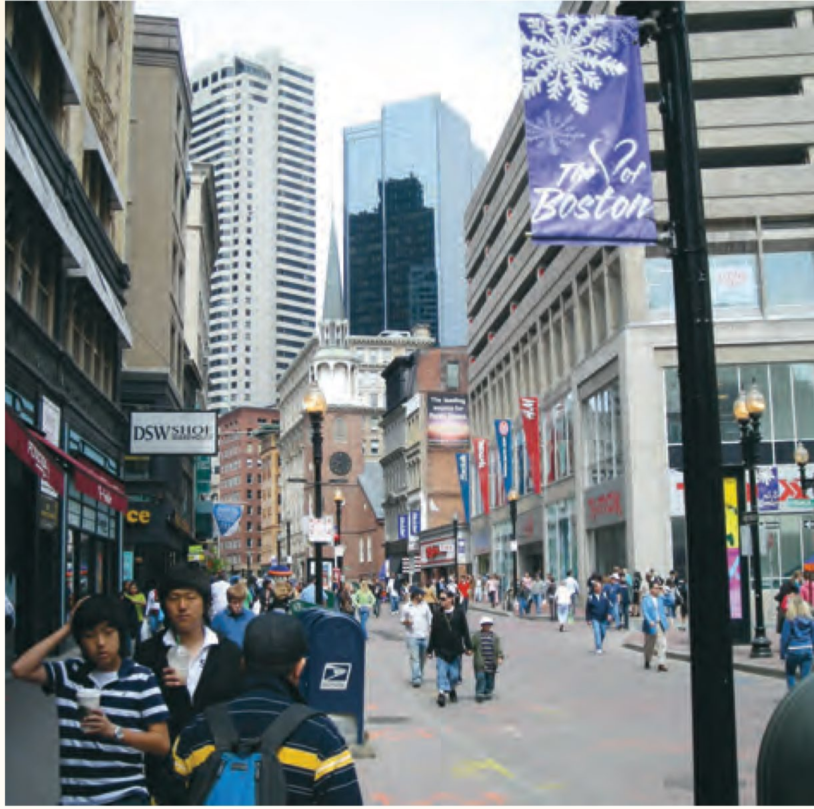
How much should Boston be willing to spend now to avoid repeated $1.4 billion costs?

For 10 years: In billions

Payments at different times have different present values:

$$PV = \sum_{t=50}^{59} 1.4e^{-0.01t} = \$8.12 \text{ bn}$$

$$PV = \int_{50}^{60} 1.4e^{-0.01t}dt = \$8.08 \text{ bn}$$



For 20 years: In billions

$$PV = \sum_{t=50}^{69} 1.4e^{-0.01t} = \$15.47 \text{ bn}$$

$$PV = \int_{50}^{70} 1.4e^{-0.01t}dt = \$15.39 \text{ bn}$$

What concerns are missed in the choice between pay now or pay later?

Note that the amounts for 20 years are **not** double the amounts for 10 years.

Adapting to Sea Level and River Rise

Flood Walls or Levees? At What Cost?

New Jersey: 2005

Flooding in the Study Area, April 2005

Cost: Varies by height above ground.

- **Levees/berms:** 2 feet: $60/linear foot; 4 feet: $106/linear foot; 6 feet: $170/linear foot
- **Floodwalls:** 2 feet: $92/linear foot; 4 feet: $140/linear foot; 6 feet: $195/linear foot

https://www.nap.usace.army.mil/Portals/39/docs/Civil/DelComp/Levees%20and%20Floodwalls.pdf

Missouri: 2015

Flood Wall

US Army Corps of Engineers floodwall in St. Louis, MO

https://www.theatlantic.com/photo/2011/05/mississippi-floodwaters-roll-south/100069/

Levee

Mississippi: 2011

1. A levee protects a home surrounded by floodwater from the Yazoo River on May 18, 2011 near Vicksburg, Mississippi. The flooded Mississippi River is forcing the Yazoo River to top its banks where the two meet near Vicksburg causing towns and farms upstream on the Yazoo to flood. #

2006 Flood

Lourdes Hospital, Binghamton, NY

http://floodbreak.com/wp-content/uploads/2012/08/FEMA_FloodMitigationBestPractice_Lourdes-Hospital_Detail-copy.pdf
https://www.tidalbasingroup.com/assets/CaseStudy/5f5596ee3f/Lady-of-Lourdes-Hospital-Flooding.pdf

- June 2006 Flood: Hospital first floor 16-20” water and raw sewage. Had to evacuate patients and close for 2 weeks; $2 million damage
- Summer 2010: Concrete 11-foot floodwalls to 500-year flood level. Cost $7 million

Analyzing the Expenditure for Lourdes Hospital Wall

- Convert all costs and benefits to the Present Value. Exponential decay
- Benefits (a.k.a. absence of damage) and some costs have to be estimated. For example: without wall, flood every five years with increasing damage, $2, $3, $4 million
- Net Present Value

$$NPV = \text{PV Benefits} - \text{PV Costs}$$

- If $NPV > 0$ project has paid off
- Observe how NPV depends on discount rate

Lourdes Hospital, NY							
Discount rate	10%						
Year	Damage/Cost (millions)	PV cost 2010	Total Cost PV	Benefit	PV benefit 2010	Total Benefit PV	NPV (millions)
2006	2	2.928			0		0.000
2007		0			0		0.000
2008		0			0		0.000
2009		0			0		0.000
2010	7	7.000	9.928		0	0.000	-9.928

Discount Rate	First Year Benefit
1%	2021
2%	2021
5%	2026
10%	2041

http://floodbreak.com/floodbreak-protects-lourdes-hospital-during-historic-2011-flood/

2011 Flood

Lourdes Hospital, NY

September 2011 Flood: Tropical storm Susquehanna river reached 11 feet above 2006. This time the hospital remained fully operational

"The floodwall itself worked magnificently, the gates operated as expected,"

The floodwaters came within 30” but never breached.

http://floodbreak.com/wp-content/uploads/2012/08/FEMA_FloodMitigationBestPractice_Lourdes-Hospital_Detail-copy.pdf
https://www.tidalbasingroup.com/assets/CaseStudy/5f5596ee3f/Lady-of-Lourdes-Hospital-Flooding.pdf

Data Driven Modeling: Covid Resources

Problems, Activities, Spreadsheets, Solutions:

- https://mcwg.github.io/covid/

Data:

- https://isaac-flath.shinyapps.io/coronavirus2/

Contact for full access to materials:

- mcwg.contact@gmail.com

References: Climate Resources

- "Projections of global-scale extreme sea levels and resulting episodic coastal flooding over the 21st century", E. Kirezci et al, 2020. www.nature.com/scientificreports
- "New elevation data triple estimates of global vulnerability to sea-level rise and coastal flooding", S. Kulp et al, 2019, https://www.nature.com/articles/s41467-019-12808-z
- https://www.climatecentral.org/news/floods-may-cost-coastal-cities-60-billion-annually-by-2050-16356
- https://www.climatecentral.org/news/report-flooded-future-global-vulnerability-to-sea-level-rise-worse-than-previously-understood
- Risk Map: https://coastal.climatecentral.org/map/
- "Enhancing Resilience in Boston: A Guide for Large Buildings and Institutions" **February2015**

Thank you!
Please get in touch!